SHEET METAL WORK

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SHEET METAL WORK

(PART II)

CALCULATIONS AND SCIENCE

For the Intermediate Examination of the
City and Guilds of London Institute

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PREFACE

This book covers the syllabus for the City and Guilds of London Institute's examination at Ordinary Level, in conjunction with *Sheet Metal Work* Part I. It deals with the calculations, science and workshop technology required, and gives the basic information on the preparation of joints and seams which is required to complete the practical test pieces successfully.

We would like to acknowledge, with thanks, the permission kindly given by the C.G.L.I. to use examples from past examination papers. Also we are indebted to Messrs. F. J. Edwards, Ltd., for the loan of many excellent photographs of sheet metal work machines and tools, and for the technical advice given by Mr. J. W. Langton, M.B.E., B.Sc., M.I.Mech.E., their general manager for North and Midlands. We would also like to thank Mr. W. R. Lewis, B.Sc. (Lond.), of the Tin Research Institute, for permission to reproduce diagrams and extracts from the text of his excellent book *Notes on Soldering*.

R. G. B.
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1958

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CALCULATIONS

A **vulgar** or **common fraction** is one such as $\frac{3}{8}$, $\frac{9}{16}$ or $\frac{11}{32}$.

The number below the line is called the **denominator** (or the namepart), that is, the number of parts into which the whole has been divided. The number above the line is called the **numerator**, and gives the number of parts taken.

Consider $\frac{3}{8}$ in. (three eighths of one inch). The inch is divided into 8 equal parts and 3 of these are present. Similarly, in $\frac{9}{16}$ in., the inch is divided into 16 equal parts and 9 are present. Fig. 1 shows the end portion of a steel rule on which one inch has been divided into sixteenths and eighths.

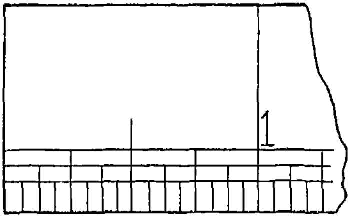


FIG. 1

In the space of $\frac{1}{2}$ inch there are two $\frac{1}{4}$ in., four $\frac{1}{8}$ in. and eight $\frac{1}{16}$ in.; it is clear therefore that these must be equal,

i.e., $\frac{1}{2}$ in. $= \frac{2}{4}$ in. $= \frac{4}{8}$ in. $= \frac{8}{16}$ in.

Notice that in each step both the numerator and the denominator have been multiplied by 2.

$$\frac{1}{2}\text{ in.} \times \frac{2}{2} = \frac{2}{4}\text{ in.}$$
$$\frac{2}{4}\text{ in.} \times \frac{2}{2} = \frac{4}{8}\text{ in.}$$
$$\frac{4}{8}\text{ in.} \times \frac{2}{2} = \frac{8}{16}\text{ in.}$$

The length has not been altered in any way, only the number concerned in the fraction.

Rule 1: When the numerator *and* denominator in any vulgar fraction are multiplied or divided by the same number, the value of the fraction is not altered.

Hence, $\frac{7}{16}$ in. has the same value as $\frac{7}{16}$ in. $\times \frac{4}{4} = \frac{28}{64}$ in.

$\frac{9}{12}$ in. has the same value as $\frac{9}{12}$ in. $\div \frac{3}{3} = \frac{3}{4}$ in.

Since it is easier to work with small numbers than large, fractions should always be reduced to their lowest terms. $\frac{12}{32}$ in. should be taken to $\frac{3}{8}$ in. by dividing both the top and bottom (numerator and denominator) by 4. This process is known as *cancelling* and will often save a good deal of work.

Mixed numbers are partly whole numbers and partly fractions, e.g., $3\frac{3}{4}$ or $5\frac{7}{8}$. In a calculation it is often necessary to change mixed numbers into fractions.

Consider the term $3\frac{2}{3}$. There are three thirds in each whole unit, therefore $3 \times 3 = 9$ thirds in three.

Now $3\frac{2}{3} = \frac{9}{3} + \frac{2}{3} = \frac{11}{3}$.

Similarly, $5\frac{7}{8}$ may be expressed as $\frac{47}{8}$.

For the purpose of transposition it may be found convenient to express a whole number as a fraction,

i.e., $$5 = \frac{5}{1}.$$

When a mixed number is changed to a ' top-heavy ' fraction, it is called an **improper fraction.**

$\frac{11}{3}$, and $\frac{47}{8}$ are **improper fractions.**

$\frac{2}{3}$, and $\frac{7}{8}$ are **proper fractions.**

The improper fractions $\frac{11}{3}$ and $\frac{47}{8}$ can be changed to mixed numbers simply by dividing the numerator by the denominator, and leaving any remainder in the form of a proper fraction.

i.e., $$\frac{11}{3} = 11 \div 3 = 3\frac{2}{3}$$
$$\frac{47}{8} = 47 \div 8 = 5\frac{7}{8}$$

Addition of Fractions

In any calculation of quantities the unit used must be the same throughout. The usual units used in sheet metal work are inches, feet, pounds, tons, gallons, etc. To add together 4 inches and 6 minutes would be meaningless. Nor could 23 inches be added to 7 feet unless they were both first expressed in the same units. For the same reason, fractions cannot be added to, or subtracted from, each other unless they have the same denominator, called a **common denominator.**

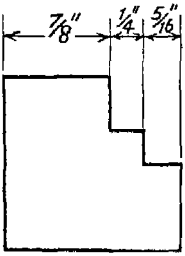


FIG. 2

EXAMPLE 1. *Calculate the total length of the template shown* (Fig. 2).

The total length will be $\frac{7}{8}$ in. + $\frac{1}{4}$ in. + $\frac{5}{16}$ in. Now $\frac{7}{8}$ in. and $\frac{1}{4}$ in. may be expressed in sixteenths by applying Rule 1.

$$\tfrac{7}{8} \text{ in.} \times \tfrac{2}{2} = \tfrac{14}{16} \text{ in.}$$

also $$\tfrac{1}{4} \text{ in.} \times \tfrac{4}{4} = \tfrac{4}{16} \text{ in.}$$

∴ rewriting, $\frac{14}{16}$ in. + $\frac{4}{16}$ in. + $\frac{5}{16}$ in. = $\frac{23}{16}$ in.

and this may be expressed as $\underline{1\frac{7}{16}}$ in.

Sometimes the common denominator is not easily recognised, but it may always be obtained by multiplying together all the denominators, for example,

$$\tfrac{1}{2} + \tfrac{2}{3} + \tfrac{3}{5}, \text{ common denominator} = 2 \times 3 \times 5 = 30.$$

Applying Rule 1, converting the fractions to thirtieths,

$$\tfrac{1}{2} \times \tfrac{15}{15} = \tfrac{15}{30}$$
$$\tfrac{2}{3} \times \tfrac{10}{10} = \tfrac{20}{30}$$
$$\tfrac{3}{5} \times \tfrac{6}{6} = \tfrac{18}{30}$$

rewriting, $\tfrac{15}{30} + \tfrac{20}{30} + \tfrac{18}{30} = \tfrac{53}{30} = 1\tfrac{23}{30}$

However, this may often produce a denominator much larger than necessary. Therefore the following method is suggested.

EXAMPLE 2. *Find the value of* $\tfrac{7}{16} + \tfrac{3}{10} + \tfrac{1}{4}$.

(*a*) Set down the denominators in a row.
(*b*) Divide by any factor common to any two or more.
(*c*) Again divide by any factor common to two or more. Notice that any number that is not wholly divisible by the factor is simply rewritten underneath.
(*d*) Multiply together the factors and the remainder, viz.

$$\begin{array}{r|rrr} 2 & 16, & 10, & 4 \\ \hline 2 & 8, & 5, & 2 \\ \hline & 4, & 5, & 1 \end{array}$$

$$2 \times 2 \times 4 \times 5 \times 1 = 80$$

Rewriting the fractions with 80 as the denominator.

$$\tfrac{7}{16} \times \tfrac{5}{5} = \tfrac{35}{80}$$
$$\tfrac{3}{10} \times \tfrac{8}{8} = \tfrac{24}{80}$$
$$\tfrac{1}{4} \times \tfrac{20}{20} = \tfrac{20}{80}$$

$$\therefore \quad \tfrac{35}{80} + \tfrac{24}{80} + \tfrac{20}{80} = \tfrac{79}{80}$$

The above is usually written over one denominator

$$\frac{35 + 24 + 20}{80} = \underline{\frac{79}{80}}$$

Subtraction of Fractions

EXAMPLE 3. *From* $\tfrac{3}{4}$ *subtract* $\tfrac{2}{5}$.

$$\text{Common denominator} = 4 \times 5 = 20$$

Convert the fractions into twentieths,

$$\tfrac{3}{4} \times \tfrac{5}{5} = \tfrac{15}{20}$$
$$\tfrac{2}{5} \times \tfrac{4}{4} = \tfrac{8}{20}$$

$$\frac{15 - 8}{20} = \frac{7}{20}$$

When subtracting mixed numbers it may be advisable first to change the mixed numbers into improper fractions. This is done when the fraction part of the number to be taken away is larger than the fraction part of the other number.

EXAMPLE 4. $9\frac{3}{8} - 6\frac{4}{5}$

$$9\tfrac{3}{8} = \tfrac{75}{8},\quad 6\tfrac{4}{5} = \tfrac{34}{5}$$

Common denominator $= 5 \times 8 = 40$

$$\tfrac{75}{8} \times \tfrac{5}{5} = \tfrac{375}{40}$$

$$\tfrac{34}{5} \times \tfrac{8}{8} = \tfrac{272}{40}$$

$$\therefore \quad \frac{375 - 272}{40} = \frac{103}{40} = \underline{2\frac{23}{40}}$$

EXAMPLE 5. $7\frac{7}{8} - 4\frac{3}{16}$.

In this case $\frac{3}{16}$ is less than $\frac{7}{8}$ and therefore can be subtracted directly.

$$7 - 4 = 3$$

$$\tfrac{7}{8} - \tfrac{3}{16} = \tfrac{14}{16} - \tfrac{3}{16} = \tfrac{11}{16}$$

$$\therefore \quad 7\tfrac{7}{8} - 4\tfrac{3}{16} = \underline{3\tfrac{11}{16}}$$

Worked Examples

1. *Four sheet metal boxes are found to weigh* $3\frac{1}{4}$ *lb*, $2\frac{7}{8}$ *lb*, $2\frac{4}{5}$ *lb*, $3\frac{1}{8}$ *lb Find the total weight.*

Total wt. $= 3\frac{1}{4} + 2\frac{7}{8} + 2\frac{4}{5} + 3\frac{1}{8}$ lb.
Adding the whole numbers $3 + 2 + 2 + 3 = 10$ lb.
Adding fractions $\frac{1}{4} + \frac{7}{8} + \frac{4}{5} + \frac{1}{8}$.
Common denominator $8 \times 5 = 40$.

$$\tfrac{1}{4} \times \tfrac{10}{10} = \tfrac{10}{40}\text{ lb.}$$

$$\tfrac{7}{8} \times \tfrac{5}{5} = \tfrac{35}{40}\text{ lb.}$$

$$\tfrac{4}{5} \times \tfrac{8}{8} = \tfrac{32}{40}\text{ lb.}$$

$$\tfrac{1}{8} \times \tfrac{5}{5} = \tfrac{5}{40}\text{ lb.}$$

$$\therefore \quad \frac{10 + 35 + 32 + 5}{40} = \frac{82}{40} = 2\frac{2}{40} = 2\frac{1}{20}$$

Total weight $= 10 + 2\frac{1}{20} = \underline{12\frac{1}{20}\text{ lb.}}$

2. *Simplify* $5\frac{3}{4} + 1\frac{7}{32} - 3\frac{3}{8}$.

Dealing first with the whole numbers, $5 + 1 - 3 = 3$,
then the fractions $\frac{3}{4} + \frac{7}{32} - \frac{3}{8}$,
Common denominator $= 32$

$$\frac{24 + 7 - 12}{32} = \frac{31 - 12}{32} = \frac{19}{32} = 3\frac{19}{32}$$

3. *A small angle bracket is to be made as shown* (Fig. 3). *If* $\frac{1}{10}$ *in. is to be added to the overall length to allow for the bend, determine the length of metal required.*

$$\begin{aligned}\text{Length required} &= 1\tfrac{3}{16}\text{ in.} + 3\tfrac{1}{8}\text{ in.} + \tfrac{1}{10}\text{ in.}\\ &= 4\text{ in.} + \tfrac{3}{16}\text{ in.} + \tfrac{1}{8}\text{ in.} + \tfrac{1}{10}\text{ in.}\\ &= 4\text{ in.} + \frac{15 + 10 + 8\text{ in.}}{80}\\ &= \underline{4\tfrac{33}{80}}\text{ in.}\end{aligned}$$

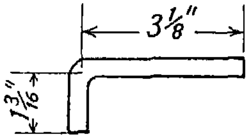


FIG. 3

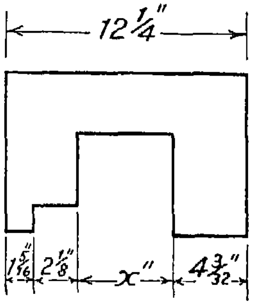


FIG. 4

4. *Calculate the length 'x' on the template shown* (Fig. 4).

$$\begin{aligned}\text{Total known lengths} &= 1\tfrac{5}{16}\text{ in.} + 2\tfrac{1}{8}\text{ in.} + 4\tfrac{3}{32}\text{ in.}\\ &= 7 + \tfrac{5}{16}\text{ in.} + \tfrac{1}{8}\text{ in.} + \tfrac{3}{32}\text{ in.}\\ &= 7 + \frac{10 + 4 + 3}{32}\\ &= 7\tfrac{17}{32}\text{ in.}\\ \text{The length } x &= \text{Overall length} - 7\tfrac{17}{32}\text{ in.}\\ &= 12\tfrac{1}{4}\text{ in.} - 7\tfrac{17}{32}\text{ in.}\\ (\text{Using } 12 = 11 + 1) &= 11\text{ in.} + 1\tfrac{1}{4}\text{ in.} - 7\tfrac{17}{32}\text{ in.}\\ &= 4\text{ in.} + \tfrac{5}{4}\text{ in.} - \tfrac{17}{32}\text{ in.}\\ &= 4\text{ in.} + \frac{40 - 17}{32}\text{ in.}\\ \text{Length } x &= \underline{4\tfrac{23}{32}}\text{ in.}\end{aligned}$$

Exercise 1

1. Express as improper fractions, $4\frac{7}{8}$, $12\frac{2}{3}$, $7\frac{3}{8}$, $3\frac{5}{16}$, $9\frac{11}{32}$, $8\frac{1}{16}$.
2. Change to mixed numbers, $\frac{45}{4}$, $\frac{29}{10}$, $\frac{13}{8}$, $\frac{27}{16}$, $\frac{81}{32}$, $\frac{75}{64}$.
3. Express as sixteenths,
   (*a*) $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$.
   (*b*) Express as eighths, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$.
   (*c*) Express as sixty-fourths, $\frac{7}{32}$, $\frac{5}{8}$, $\frac{13}{16}$, $\frac{7}{8}$.
4. Express in their lowest terms, $\frac{4}{8}$, $\frac{12}{16}$, $\frac{5}{10}$, $\frac{24}{32}$, $\frac{14}{64}$, $\frac{10}{16}$.
5. Add, $\frac{3}{4} + \frac{7}{16}$, $\frac{3}{8} + \frac{7}{32}$, $\frac{5}{16} + \frac{9}{64}$, $\frac{1}{4} + \frac{3}{16} + \frac{7}{16}$, $\frac{3}{4} + \frac{7}{16} + \frac{1}{8}$, $\frac{3}{4} + \frac{5}{16} + \frac{7}{8}$.

6. Subtract, $\frac{7}{8}-\frac{1}{2}$, $\frac{29}{32}-\frac{17}{64}$, $\frac{1}{2}-\frac{1}{16}$, $\frac{7}{8}-\frac{1}{16}$, $\frac{5}{8}-\frac{3}{16}$, $1-\frac{1}{8}$, $2-\frac{3}{4}$, $7-1\frac{1}{8}$, $9-3\frac{3}{8}$, $10-4\frac{5}{16}$.
7. Simplify, (*a*) $\frac{3}{4}+\frac{1}{8}-\frac{5}{16}$, (*b*) $\frac{1}{2}+\frac{1}{16}-\frac{3}{8}$, (*c*) $\frac{7}{8}+\frac{5}{16}-\frac{5}{32}$, (*d*) $1\frac{3}{8}+1\frac{5}{8}-1\frac{7}{16}$, (*e*) $\frac{7}{8}-\frac{3}{8}+\frac{1}{4}$, (*f*) $\frac{5}{16}-\frac{1}{8}+\frac{3}{4}$, (*g*) $1\frac{3}{4}-\frac{3}{16}+\frac{5}{8}$, (*h*) $2\frac{9}{16}-\frac{7}{8}+\frac{5}{32}$, (*i*) $5\frac{3}{8}-\frac{3}{4}+\frac{3}{16}$.
8. Simplify, (*a*) $4\frac{7}{8}-\frac{1}{16}-\frac{5}{8}$, (*b*) $5\frac{3}{4}-\frac{1}{2}-\frac{11}{16}$, (*c*) $6\frac{1}{8}-1\frac{7}{8}-\frac{3}{16}$, (*d*) $9\frac{3}{16}-2\frac{5}{8}-1\frac{3}{16}$, (*e*) $14-4\frac{3}{8}-\frac{7}{16}$, (*f*) $8-3\frac{5}{16}-\frac{3}{4}$.
9. 3 inches are cut from a 12 in. length of wire. What fraction is cut off?
10. 8 inches are cut from a 24 in. length of wire. What fraction remains?
11. How much must be added to $\frac{7}{8}$ lb. of solder to make $2\frac{3}{4}$ lb?

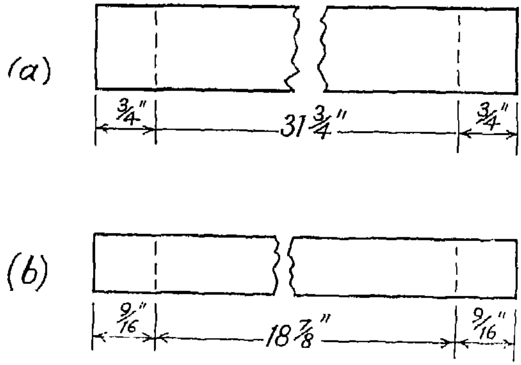


FIG. 5

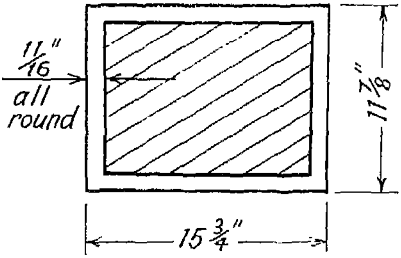


FIG. 6

12. $\frac{3}{4}$ lb of solder is taken from a $10\frac{1}{2}$ lb tin, how much remains?
13. Calculate the total lengths required for the patterns shown (Fig. 5).
14. The outside diameter of a pipe is $3\frac{5}{8}$ in. The metal thickness is $\frac{5}{16}$ in., calculate the inner diameter.
15. $3\frac{7}{8}$ in. is cut from a length of piping, then $7\frac{1}{2}$ in., how much remains if the pipe was originally $18\frac{3}{4}$ in. long?
16. A length of flat steel strip is $36\frac{1}{2}$ in. long. If $\frac{15}{16}$ in. is to be trimmed from each end to clean up the strip, how much will remain?
17. Two lengths of brass sheet $4\frac{5}{8}$ in. and $5\frac{1}{2}$ in. long are to be soldered together with an overlap of $\frac{3}{4}$ in. How long will the soldered length be?
18. Calculate the inside length and breadth of the shaded space (Fig. 6).

19. Calculate the overall length and breadth of the box pattern shown (Fig. 7).

20. A man's clock card shows he worked on a job: Tues. $6\frac{3}{4}$ hrs, Wed. $8\frac{1}{2}$ hrs, Thurs. $5\frac{1}{4}$ hrs. If on Wednesday he had $\frac{3}{4}$ hr waiting time, what was his total time on the job?

21. A man uses $1\frac{1}{8}$ in. from a stick of solder on a job, $3\frac{3}{4}$ in. on another, and $2\frac{7}{16}$ in. on another. What length of solder will remain, if the original length of the stick was $10\frac{1}{4}$ in.?

22. For a certain job 38 lengths of steel tube $13\frac{1}{16}$ in. long are required, also 27 lengths $8\frac{1}{2}$ in. long. What is the total length of tubing required?

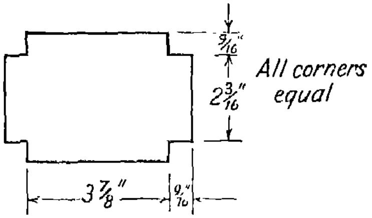


FIG. 7

23. Calculate the length of wire required to wire the top of a rectangular box $6\frac{5}{8}$ in. long × $3\frac{7}{16}$ in. wide.

Multiplication of Fractions

If 4 is multiplied by 5 the result is 20, written $4 \times 5 = 20$ or 4 of $5 = 20$. The word 'of' has the same meaning as the multiplication sign ×.

If one half is multiplied by three quarters, then

$$\tfrac{1}{2} \times \tfrac{3}{4} \text{ or } \tfrac{1}{2} \text{ of } \tfrac{3}{4} = \tfrac{3}{8}.$$

The value of $\frac{3}{8}$ has been obtained by multiplying the two numerators together for the final numerator, and the two denominators together for the final denominator,

that is,
$$\tfrac{1}{2} \times \tfrac{3}{4} = \frac{1 \times 3}{2 \times 4} = \tfrac{3}{8}.$$

EXAMPLE 1. *Multiply* $6\frac{1}{4} \times 1\frac{3}{4}$

$$= \tfrac{25}{4} \times \tfrac{7}{4} = \tfrac{175}{16} = 10\tfrac{15}{16}.$$

If any numerator and denominator contain a common factor, they may be divided by this factor. Thus, in the following example, both 3 and 2 are common factors and may be used for dividing.

$$\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} = \frac{1 \times 1}{1 \times 2} = \frac{1}{2}$$

In this way the work has been reduced and the process is known as *cancelling*.

EXAMPLE 2. *Multiply* $\frac{7}{8} \times \frac{5}{6} \times \frac{9}{10}$.

Cancelling the 9 and 6 by 3 and the 5 and 10 by 5,

$$\frac{7}{8} \times \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{6}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{10}}} = \frac{7 \times 1 \times 3}{8 \times 2 \times 2} = \frac{21}{32}$$

Division of Fractions

The process of division is carried out to find how many times one quantity may be contained in another

$$12 \div 3 = 4.$$

Thus there are 4 threes in 12. To divide a fraction by a whole number, the numerator is simply divided by that number,

i.e., $6\frac{1}{4} \div 5 = \frac{25}{4} \div 5 = \frac{5}{4} = 1\frac{1}{4}.$

But the same result could have been obtained by multiplying the denominator by the dividing number,

$$6\tfrac{1}{4} \div 5 = \frac{25}{4 \times 5} = \frac{25}{20} = 1\tfrac{1}{4}.$$

Now consider $6 \div \frac{2}{3}$. It is known that 6 whole units are made up of 18 thirds, and also that there is one half of that number of two thirds in 6,

so that $6 \div \frac{2}{3} = 9,$

but $6 \times \frac{3}{2} = 9.$

Hence the rule for division is: Invert the divisor and multiply by the fraction thus obtained.

EXAMPLE 3. *A sheet of steel 6 ft long is to be cut into strips each ¾ in. wide. How many strips may be cut from the sheet if 3 in. of the length is waste material?*

Now, 6 ft = 6 × 12 = 72 in.

Available length = 72 − 3 = 69 in.

$$\text{Total number of strips} = 69 \div \tfrac{3}{4} = \frac{\overset{23}{\cancel{69}} \times 4}{\cancel{3}} = 92.$$

Combined Multiplication and Division

EXAMPLE 4. $4\frac{1}{2} \times 2\frac{7}{8} \div 2\frac{1}{4}$.

$$\text{Rewriting} \qquad \frac{\overset{1}{\cancel{9}}}{\underset{1}{\cancel{2}}} \times \frac{23}{\underset{4}{\cancel{8}}} \times \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{9}}} = \frac{1 \times 23 \times 1}{1 \times 4 \times 1} = \frac{23}{4} = 5\tfrac{3}{4}.$$

The order of signs

Some calculations involve the use of mixed signs +, of, −, ×, ÷, and ( ). Any term enclosed in brackets is considered to be separate from the others and should be worked out first. The correct order of dealing with signs is: brackets, of, division, multiplication, addition and subtraction. Students will find help in remembering this order from the word B.O.D.M.A.S. which is made up of the first letter of each sign.

EXAMPLE 5. *Simplify* $2\frac{1}{2} \div \frac{1}{3} + \frac{3}{4} - \frac{1}{4}$ *of* $\frac{2}{7}$.

1st $$\frac{1}{\underset{2}{\cancel{4}}} \times \frac{\overset{1}{\cancel{2}}}{7} = \frac{1}{14}$$

2nd $$2\frac{1}{2} \div \frac{1}{3} = \frac{5}{2} \times \frac{3}{1}$$
$$= \frac{15}{2}$$

Rewriting $\frac{15}{2} + \frac{3}{4} - \frac{1}{14}$

$$= \frac{210 + 21 - 2}{28} = \frac{229}{28} = \underline{8\tfrac{5}{28}}$$

EXAMPLE 6. *Determine the value of*

$$\frac{10\frac{3}{4} - 7\frac{1}{16} + 2\frac{1}{2}}{2\frac{3}{8} + (\frac{1}{4} \times \frac{7}{8})}$$

Simplifying the top line $10\frac{3}{4} - 7\frac{1}{16} + 2\frac{1}{2}$

$$= 5 + \frac{3}{4} - \frac{1}{16} + \frac{1}{2}$$
$$= 5 + \frac{12 - 1 + 8}{16}$$
$$= 5 + \frac{19}{16} = 6\frac{3}{16}$$

Now simplify the bottom line $2\frac{3}{8} + (\frac{1}{4} \times \frac{7}{8})$

$$= 2\frac{3}{8} + \frac{7}{32}$$
$$= 2 + \frac{12 + 7}{32}$$
$$= 2\frac{19}{32}.$$

Rewriting the expression $\dfrac{6\frac{3}{16}}{2\frac{19}{32}} = \dfrac{\frac{99}{16}}{\frac{83}{32}}$

$$= \frac{99}{\underset{1}{\cancel{16}}} \times \frac{\overset{2}{\cancel{32}}}{83}$$

$$= \frac{99 \times 2}{83} = \frac{198}{83} = 2\frac{32}{83}$$

$$\therefore \quad \frac{10\frac{3}{4} - 7\frac{1}{16} + 2\frac{1}{2}}{2\frac{3}{8} + (\frac{1}{4} \times \frac{7}{8})} = \underline{2\frac{32}{83}}$$

EXAMPLE 7. *1 lb of an alloy consists of $\frac{3}{4}$ lb of copper, $\frac{3}{16}$ lb of zinc and the remainder tin. Calculate how much of each metal is containied in 60 lb of the alloy.*

There must be $1 - (\frac{3}{4} + \frac{3}{16})$ lb of tin per lb of alloy

$$= 1 - \left(\frac{12 + 3}{16}\right) = 1 - \tfrac{15}{16}$$

$$= \tfrac{1}{16} \text{ lb of tin.}$$

In 60 lb there will be 60 times the amount of each metal.

$$\text{Copper} \quad \overset{15}{\cancel{60}} \times \frac{3}{\underset{1}{\cancel{4}}} = 45 \text{ lb}$$

$$\text{Zinc} \quad \overset{15}{\cancel{60}} \times \frac{3}{\underset{4}{\cancel{16}}} = \frac{45}{4} = 11\tfrac{1}{4} \text{ lb}$$

$$\text{Tin} \quad \overset{15}{\cancel{60}} \times \frac{1}{\underset{4}{\cancel{16}}} = \frac{15}{4} = 3\tfrac{3}{4} \text{ lb}$$

$$\text{Total} = \underline{\overline{60}} \text{ lb.}$$

EXAMPLE 8. *The total cost of an article is made up of: materials $\frac{1}{8}$, labour $\frac{1}{2}$, overheads $\frac{1}{8}$, profit $\frac{1}{4}$. Find the cost of each, if the article is sold for £8 10s.*

$$\begin{aligned}
\text{Materials} &= \tfrac{1}{8} \text{ of } £8\tfrac{1}{2} = \tfrac{1}{8} \times \tfrac{17}{2} = \tfrac{17}{16} = £1\tfrac{1}{16} \\
&= £1\ 1s.\ 3d. \\
\text{Overheads} &= \tfrac{1}{8} \text{ of } £8\tfrac{1}{2} = £1\ 1s.\ 3d. \\
\text{Labour} &= \tfrac{1}{2} \text{ of } £8\tfrac{1}{2} = £4\ 5s.\ 0d. \\
\text{Profit} &= \tfrac{1}{4} \text{ of } £8\tfrac{1}{2} = \tfrac{1}{4} \times \tfrac{17}{2} = \tfrac{17}{8} = £2\tfrac{1}{8} \\
&= \underline{£2\ 2s.\ 6d.}
\end{aligned}$$

Exercise 2

1. Simplify, (*a*) $\frac{7}{16} \times \frac{3}{4} \times \frac{1}{2}$; (*b*) $7\frac{3}{4} \div \frac{9}{16}$; (*c*) $8 \times 1\frac{3}{16} \div \frac{3}{8}$.
2. Simplify, (*a*) $12\frac{5}{8} \div 3\frac{1}{4}$; (*b*) $3\frac{7}{8} \div \frac{5}{16} \times 4\frac{3}{32}$; (*c*) $\frac{7}{8} \times \frac{9}{16} \div 4\frac{1}{2}$.
3. Simplify, (*a*) $\frac{9}{16} + \frac{7}{8} \times \frac{1}{4}$; (*b*) $1\frac{5}{32} \div \frac{3}{32} - \frac{3}{16}$; (*c*) $3\frac{3}{4} \div 1\frac{1}{8} + 5\frac{9}{10}$.
4. Simplify, (*a*) $3\frac{9}{32} \times \frac{7}{16} \div 2\frac{1}{4}$; (*b*) $12\frac{1}{2} + \frac{9}{64} \div \frac{3}{8}$; (*c*) $15\frac{3}{4} - 1\frac{7}{8} \times 3\frac{1}{2}$.
5. Determine the value of $\dfrac{12\frac{1}{2} - 7\frac{1}{16} + 3\frac{1}{4}}{2\frac{1}{8} + (\frac{1}{3} \text{ of } \frac{5}{8})}$.
6. Determine the value of $4\frac{3}{10} \div (7\frac{1}{6} \times \frac{4}{5})$.
7. Determine the value of $19\frac{3}{5} \div 4\frac{1}{12} \times 4\frac{1}{6}$.
8. Determine the value of $\dfrac{3\frac{1}{8} \text{ of } \frac{4}{25}}{2\frac{1}{3} + \frac{1}{2}}$.
9. Simplify $\dfrac{\frac{1}{2} + \frac{3}{8} + \frac{5}{8}}{2\frac{1}{2} + \frac{4}{5} + \frac{1}{8}}$.
10. Simplify $\frac{1}{16} \times 1\frac{5}{8} \times \dfrac{\frac{1}{3} + \frac{1}{4} + \frac{1}{12}}{\frac{1}{16} + \frac{1}{8} + \frac{3}{4}}$.
11. 25 strips each $\frac{5}{8}$ in. wide and 15 strips $\frac{3}{4}$ in. wide are cut from a sheet. Calculate the original length of the sheet if $5\frac{1}{8}$ in. remains after cutting.

12. The overlap of a riveted joint is to be 3 times the rivet diameter. What would be the overlap if $\frac{3}{16}$ in. dia. rivets were used?
13. The circumference or length around a circle is $3\frac{1}{7}$ times the diameter. Calculate the circumference of a $9\frac{7}{32}$ dia. circle.
14. Two lengths of wire each $15\frac{7}{8}$ in. long are required to wire the top of a metal box. If they are to be cut from a piece of wire $37\frac{1}{2}$ in. long, calculate the length of scrap.
15. A bonus of $\frac{2}{3}$ of the basic rate is earned on a job. If the time on the job was $15\frac{3}{4}$ hours and the rate was 4*s*. 3*d*. per hr, how much bonus was earned?
16. Two men and a boy earn between them £16 10*s*. bonus. If each man gets $\frac{3}{7}$ and the boy gets $\frac{1}{7}$, how much bonus does each receive?
17. If wire is supplied in 6-ft. lengths and from each length pieces of $12\frac{1}{4}$ in., $10\frac{1}{2}$ in., $12\frac{5}{8}$ in. and $32\frac{3}{8}$ in. are cut, determine the total length of scrap material when 55 lengths are used.
18. Calculate the length of wire required to wire the tops of 150 boxes each $9\frac{1}{2}$ in. diameter if the circumference of the box is $3\frac{1}{7}$ times the diameter.
19. 1 lb of an alloy contains $\frac{5}{8}$ lb of copper, $\frac{3}{10}$ lb of zinc and $\frac{3}{40}$ lb of tin. Calculate how much of each metal is contained in $46\frac{3}{4}$ lb of the alloy.

Decimal Fractions

The chief objection to working in vulgar fractions is that the denominators often become large and difficult to manipulate. How much easier it would be if only denominators of 1, 10, 100, 1000, etc., were used. The decimal system makes use of these simple denominators, and to mark the boundary between figures representing whole units and figures representing the fractions of a unit a small dot or point is placed immediately after the last figure representing whole units. This mark is known as the **decimal point.**

$$3 \text{ is written } 3$$
$$\frac{3}{10} \text{ is written } 0{\cdot}3$$
$$\frac{3}{100} \text{ is written } 0{\cdot}03$$
$$\frac{3}{1000} \text{ is written } 0{\cdot}003$$

The following table shows how decimal numbers are built up. Consider the number 3794·375

Thousands	Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths
3	7	9	4	·	3	7	5

Addition and subtraction are done in the same way as with whole numbers, but great care should be taken to ensure that figures of the same place value are added, and that the decimal points are placed exactly under each other.

Example 1. *Add* 64·31, 280, ·629, 17·03.

Arranging the figures in position,

$$\begin{array}{rl} 64 & \cdot 31 \\ 280 & \\ & \cdot 629 \\ 17 & \cdot 03 \\ \hline 361 & \cdot 969 \\ \hline \end{array}$$

Adding,

Example 2. *Subtract* 435·15 *from* 547·3.

$$\begin{array}{rl} 547 & \cdot 3 \\ -435 & \cdot 15 \\ \hline 112 & \cdot 15 \\ \hline \end{array}$$

To Convert Decimals to Vulgar Fractions

From previous work it follows that:

$$\begin{aligned} \cdot 5 &= \tfrac{5}{10} = \tfrac{1}{2} \\ \cdot 75 &= \tfrac{7}{10} + \tfrac{5}{100} = \tfrac{75}{100} = \tfrac{3}{4} \\ 5\cdot 25 &= 5 + \tfrac{2}{10} + \tfrac{5}{100} = 5\tfrac{25}{100} = 5\tfrac{1}{4} \end{aligned}$$

The numerator of the fraction is the figures following the decimal point, the denominator is 1 followed by as many zeros as there are figures after the point.

To Convert Vulgar Fractions to Decimals

$\tfrac{7}{16}$ is equivalent to 7 ÷ 16. If this division is carried out the decimal fraction will be obtained.

Hence,

$$\begin{array}{rl} & 0\cdot 4375 \\ 16) & \overline{7\cdot 0000(} \\ & 6\,4 \\ \hline & \;\;60 \\ & \;\;48 \\ \hline & \;\;120 \\ & \;\;112 \\ \hline & \;\;\;\;80 \\ & \;\;\;\;80 \\ \hline & \;\;\;\;.\,. \\ \hline \end{array}$$

16 will not divide into 7, put a zero above to show this. To continue, go past the decimal point to 7·0. Place the point in position after the zero in the answer and divide 16 into 70; bring down zeros as required until the answer is obtained.

$\therefore\ \tfrac{7}{16} = \underline{0\cdot 4375.}$

Indefinite Decimal Equivalents

Many vulgar fractions cannot be expressed exactly as decimals.

$$\begin{array}{r} 0{\cdot}333333\ldots\ldots. \\ \text{Consider } \tfrac{1}{3} = 1 \div 3 = 3\overline{)1{\cdot}000000}\ldots\ldots. \end{array}$$

$$\begin{array}{r} 0{\cdot}1428571428\ldots\ldots. \\ \tfrac{1}{7} = 1 \div 7 = 7\overline{)1{\cdot}0000000000}. \end{array}$$

Indefinite decimal fractions are obtained when the denominator of the vulgar fraction has prime factors other than 2 or 5.

These are called **recurring decimals** and are written with a dot above the recurring number, i.e.,

$$\tfrac{1}{3} = 0{\cdot}333\dot{3}$$

$$\tfrac{1}{7} = 0{\cdot}\dot{1}4285\dot{7}$$

$$\tfrac{1}{6} = 0{\cdot}166\dot{6}$$

In practical calculations the number of figures after the decimal point depends upon the accuracy required, which in turn depends upon the accuracy of the information given. It is an easy matter with a steel rule to measure to $\frac{1}{16}$ in., but $\frac{1}{16}$ in. = ·0625 in., and it would be foolish to suppose that you could measure to the last figure ·0005 in., which is $\frac{5}{10000}$ in. Even the third decimal figure ·002 in. is $\frac{2}{1000}$ in. or 2 ‘ thou ’, and cannot be measured on a rule.

If, however, a micrometer is used, the 2 *thou* could be measured and the $\frac{5}{10000}$ estimated. Thus, using a steel rule, the last two figures of ·0025 would not be **significant** or mean much, but with a micrometer they would.

The answer to any problem should be given in *significant figures* only, but any figures beyond these should not just be ignored but taken into account by approximating their value as follows.

The rule is: *If the figure to be dropped is* 5 *or more the figure in front of it is increased by one, whilst if it is* 4 *or less the figure is just dropped. Thus* 4·26 *would become* 4·3 *to one decimal place, but* 4·24 *would become* 4·2. *For accurate calculations always work to at least one figure beyond the number of significant figures required.*

Whole numbers are, of course, significant figures; in 3·14 there are three significant figures.

EXAMPLE 3. *Change $\frac{3}{7}$ to a decimal correct to 3 places of decimals.*

$$\begin{array}{r} \underline{0{\cdot}4285} \\ 7\overline{)3{\cdot}0000} \end{array}$$

Since the fourth figure in the answer 0·4285 is 5, when it is dropped 1 must be added to the previous figure 8.

$$\tfrac{3}{7} = 0{\cdot}429 \text{ correct to 3 places.}$$

Thus, 6·93738 = 7 correct to the nearest whole number.
= 6·9 correct to one decimal place.
= 6·94 correct to two decimal places.
= 6·937 correct to three decimal places.
= 6·9374 correct to four decimal places.

Multiplication of Decimals

Rule: *Consider the problem to be one of multiplication of whole numbers and obtain the product. Count the total number of figures to the right of the two decimal points, then count that total number of figures from the right of the answer and insert the decimal point.*

EXAMPLE 4. 8·3 × 3·14.

$$\begin{array}{r} 314 \\ \underline{83} \\ 942 \\ 2512 \\ \hline 26062 \end{array}$$

Total number of figures after the decimal points = 1 + 2 = 3. The decimal point will be placed before the 0.

$\therefore$ 8·3 × 3·14 = 26·062

EXAMPLE 5. 8·3 × ·06 × 1·1.

$$\begin{array}{r} 83 \\ \underline{06} \\ 498 \\ \underline{11} \\ 498 \\ \underline{498} \\ 5478 \end{array}$$

Total number of figures after the decimal point = 1 + 2 + 1 = 4.

$\therefore$ 8·3 × ·06 × 1·1 = 0·5478

Division of Decimals

Rule: *If the divisor (the number to be divided into the other) is a whole number, division is done as in ordinary long division. The decimal point is inserted in the answer after the first decimal figure is brought down for division.*

EXAMPLE 6. 232·68 ÷ 84.

$$\begin{array}{r@{}l} & 2\cdot77 \\ 84) & \overline{232\cdot68} \\ & 168 \\ \hline & 646 \\ & 588 \\ \hline & 588 \\ & 588 \\ \hline & \ldots \end{array} \qquad \therefore\ 232\cdot68 \div 84 = \underline{2\cdot77}$$

If the divisor contains a decimal, it should first be changed to a whole number by moving the decimal point to the right. At the same time the decimal point in the dividend should be moved an equal number of places, that is, the divisor and dividend are both multiplied by the same number without altering the value of the result.

EXAMPLE 7. 1·85472 ÷ ·504

now 1·85472 ÷ ·504 = 1854·72 ÷ 504

$$\begin{array}{r@{}l} & 3\cdot68 \\ 504) & \overline{1854\cdot72} \\ & 1512 \\ \hline & 3427 \\ & 3024 \\ \hline & 4032 \\ & 4032 \\ \hline & \ldots\ldots \end{array} \qquad \therefore\ 1\cdot85472 \div \cdot504 = \underline{3\cdot68}$$

Practical Examples

1. *The length of a sheet required to be rolled into a cylinder is found by multiplying the diameter by* 3·14. *Calculate the length of metal required to make a cylinder* 4·75 *in. diameter.*

Length of metal = 4·75 in. × 3·14

$$\begin{array}{r} 475 \\ 314 \\ \hline 1900 \\ 475 \\ 1425 \\ \hline 149150 \text{ in.} \\ \hline \end{array}$$

Total number of figures after decimal points = 2 + 2 = 4

∴ Length of metal = 14·9150 in.

To 3 significant figures = 14·9 in.

2. *The total length of wire required for a box top is* 16·4 *in. How many boxes could be completely wired from a coil* 100 *yards long, ignoring scrap?*

$$\begin{aligned}\text{Now, 1 yard} &= 36 \text{ in.}\\ \therefore\ \text{100 yards} &= 3{,}600 \text{ in.}\\ \text{Number of lengths of wire} &= 3{,}600 \div 16{\cdot}4\\ &= 36{,}000 \div 164\end{aligned}$$

$$\begin{array}{r} 219\\ =164\overline{)36000}\\ 328\\ \hline 320\\ 164\\ \hline 1560\\ 1476\\ \hline 84\\ \hline \end{array}$$

$\therefore$ Total complete lengths $= 219$

$+$ Scrap wire $= \frac{84}{164} = \frac{21}{41}$ of a length.

$\therefore$ A coil of wire 100 yards long would wire completely 219 boxes.

Exercise 3

1. Multiply (*a*) 62·3 × ·625; (*b*) 24·13 × 2·491.
2. Divide (*a*) 19·95 by 1·05; (*b*) 0·00384 by 0·00032; (*c*) 22·876 by 0·00076.
3. Evaluate (*a*) $\frac{0{\cdot}003 \times {\cdot}004}{0{\cdot}005}$; (*b*) $\frac{41{\cdot}82 \times 166{\cdot}8}{6{\cdot}273}$; (*c*) $\frac{0{\cdot}026 \times 1{\cdot}87 \times 0{\cdot}0042}{0{\cdot}7 \times 6{\cdot}63 \times 5{\cdot}5}$
4. Convert to decimal form $\frac{5}{64}$, $11\frac{9}{32}$, $7\frac{9}{10}$, $\frac{47}{64}$.
5. Rewrite in decimal form $5\frac{5}{16}$, $9\frac{1}{32}$, $6\frac{3}{8}$, $3\frac{1}{7}$.
6. If a rod expands 0·000012 in. for 1 degree rise in temperature, how much will it expand for a rise in temperature of (*a*) 100 degrees, (*b*) 87 degrees, (*c*) 157·5 degrees?
7. Aluminium sheet weighs 0·096 lb per cu. in. Calculate the weight of 480 cu. in.
8. 16 gauge wire weighs 0·017 lb per ft. Determine the weight of 150 yards of this wire.
9. If the wire required for the top of a cylinder is 18·28 in. long, calculate the weight of wire used when wiring 180 tops. 1 ft length of wire weighs 0·02 lb.
10. A guillotine cuts strips from a continuous rolling mill at the rate of 12 strips per minute. If each strip is 15·4 in. wide, determine (*a*) How many ft of metal will be used in 5 hr; (*b*) The total number of strips cut in this time.
11. Wire is bought in straight lengths each 6 ft long, and then cut into pieces 5·785 in. long. If 192 of these pieces are required, determine (*a*) the length of scrap wire on each length; (*b*) the total number of lengths of wire required; (*c*) the overall length of scrap wire.
12. In marking out the patterns for a hopper 12 sq. ft of metal was used, and of this amount 1·86 sq. ft was cut away in notching, wiring, waste, etc. If the metal weighs 4·27 lb per sq. ft, determine the weight of 52 hoppers.

Ratio

When two quantities are compared in terms of length, weight, cost, etc., the comparison is called a **ratio.**

If one job weighs twice as much as another, the ratio is 2 : 1 the sign : meaning '*as to*'. A ratio can be written as 2 : 1, or in fraction form $\frac{2}{1}$.

The comparison or ratio can only be made between two quantities having the same unit. We could not compare feet : pounds or £ : inches.

One common use of a ratio is in scale drawings. A drawing may be covered by a note 'Scale: $\frac{1}{4}$ size' or 'Scale: 1 : 8'. On the first drawing the ratio of any length drawn to the actual length on the part is 1 : 4 or $\frac{1}{4}$. A length of 4 in. on the part would be drawn 1 in. long, 12 in. on the part would be drawn 3 in. long.

EXAMPLE 1. *A man and a third-year apprentice work on a job. The bonuses they earn is shared between them in the ratio of* 5 : 2; *if the bonus amounts to* £1 15*s. how much should each get?*

The man gets 5 parts of the bonus, the boy gets 2 parts.

$\therefore$ Total number of parts $= 5 + 2 = 7$.

With 7 parts each part would be £1 15*s.* ÷ 7.

1 part = £1 15*s.* ÷ 7 = 35*s.* ÷ 7 = 5*s.*
and the man's share = 5 × 5*s.* = $\underline{25s.}$
the boy's = 2 × 5*s.* = $\underline{10s.}$

(Check: shares added = 25*s.* + 10*s.* = 35*s.* = £1 15*s.*)

It will be seen that the method is: (*a*) find the total number of parts making up the ratio, (*b*) calculate the value of one of these parts and then (*c*) divide up to the value of the ratio.

EXAMPLE 2. *A* 4½ *ft length of rod has to be cut up in the ratio of* 5 : 4. *Find the length of each part.*

Total number of parts $= 5 + 4 = 9$
One part = 4½ ft ÷ 9 = ½ ft
4 parts = $4 \times \frac{1}{2} = \underline{2\text{ ft}}$ 5 parts = $5 \times \frac{1}{2} = \underline{2\frac{1}{2}\text{ ft}}$
(Check: Total = 2 + 2½ = 4½ ft.)

EXAMPLE 3. *A solder is made up of tin, lead and antimony in the ratio of* 19 : 30 : 1 *by weight. What weight of each is needed to make up* 75 *lb of solder?*

Total number of parts $= 19 + 30 + 1 = 50$.
Weight of tin $= \frac{19}{50}$ of 75 $= \frac{57}{2} = 28\frac{1}{2}$ lb.
Weight of lead $= \frac{30}{50}$ of 75 $= 45$ lb.
Weight of antimony $= \frac{1}{50}$ of 75 $= 1\frac{1}{2}$ lb.
(Check: Total $= 28\frac{1}{2} + 45 + 1\frac{1}{2} = 75$ lb.)

Proportion

Consider a 60 degree set square as used in drawing work (Fig. 8). In the set square ABC, or triangle ABC, if the length of AC was increased to E the lengths of AB and CB would increase *in proportion* to make the new triangle ADE.

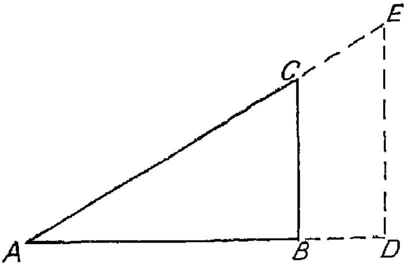


FIG. 8

In any 60° set square the length of AC is twice the length BC. Likewise the length of AE is twice the length DE. That is, the ratio of these two sides is always 2 : 1.

Proportion Problems

EXAMPLE 4. *A certain size of rivet costs* 8*s*. 6*d*. *per gross. If* 480 *of these rivets are used in making a tank, how much would these rivets cost?*

As the number of rivets increases so does the cost.

144 cost 8*s*. 6*d*. or 102 pence
1 costs $\frac{102}{144}$ pence
480 cost $\frac{102}{144} \times 480 = 340d. = £1\ 8s.\ 4d.$

This method is called the *unitary method*. First the cost of one is found and this is multiplied by the number required.

Another, and probably more simple, method is the *fractional method*.

102*d*. for 144 rivets
xd. for 480 rivets (writing x to mean how many?)
$x = 102 \times \frac{480}{144} = 340d. = £1\ 8s.\ 4d.$

Using this method the *nature* of the proportion must be known. With *direct proportion*, as one value increases so does the other, if one decreases so does the other. The above problem was one of direct proportion.

With *indirect proportion*, as one value increases the other decreases. A simple example of indirect proportion arises in sport; the faster an athlete runs the shorter the time he can keep up his pace. The 100 yards is run *all out*, the mile is run much slower because speed and endurance are in indirect proportion to each other.

EXAMPLE 5. *If* 12 *men take* 8 *days to complete a certain job how long should it take* 15 *men to do it?*

8 days with 12 men

x days with 15 men

The time will be less.

$$\therefore \quad x = 8 \times \tfrac{12}{15} = \underline{6\tfrac{2}{5}\text{ days.}}$$

The nature of the proportion must be known. The time will vary with the ratio of either $\frac{15}{12}$ or $\frac{12}{15}$. Since it is known that the time should be less the correct ratio to apply must be $\frac{12}{15}$.

EXAMPLE 6. *A rectangular pattern is* 30 *in. long and* 9 *in. broad. The drawing is scaled down to reduce the amount of paper required for it. On the drawing the length is drawn* 5 *in. long. What should be the breadth?*

5 in. represents 30 in.

x in. represents 9 in.

This is direct proportion—the breadth will also be less.

$$x = 5 \times \tfrac{9}{30} = \underline{1\tfrac{1}{2}\text{ in.}}$$

EXAMPLE 7. *When ordering material a storekeeper must consider the rate at which it is used up. If* 87 *men are working, a packet of rivets lasts* 26 *days. How long will this packet last if* 58 *men are employed?*

26 days with 87 men

x days with 58 men

The proportion is indirect—the packet will last longer

$$x = 26 \times \tfrac{87}{58} = 39\text{ days.}$$

Exercise 4

1. A bonus of £5 12*s*. has to be shared between two workers in the ratio of 5 : 2. How much does each receive?
2. A template is to be rectangular in shape, and 30 in. long and 9 in. broad. When it is drawn to scale the length is 4 in. What should be the breadth when drawn?
3. A main outlet pipe has an effective area of 100 sq. in. Determine the areas of 3 junction pipes if they are in ratios of 1 : 3 : 4.
4. 5½ yards of pipe cost 8*s*. 3*d*. Find the price of 3¾ yards.
5. If 3 cwt 28 lb of brazing alloy cost £12 5*s*. 0*d*., what will be the cost of 5 cwt 56 lb?
6. 144 yards of wire are bought for £5 8*s*. 0*d*. If 252 ft are used on a job, what is the value of the remainder?
7. A man and two apprentices work on a job and share the bonus in ratios of 6 : 4 : 3. If £8 9*s*. 0*d*. is earned, how much does each receive?

8. When 27 men are working on a production line of presses, 2619 metal blanks are used each day. If 8 men are stopped whilst maintenance is carried out, what will be the daily reduction of output?
9. A quantity of metal bowls were made by hand at a cost of 7*s.* each. When the order was repeated it was decided to make a press tool at the cost of £56 and reduce the cost to 2*s.* 4*d.* per bowl. Which is the cheaper method of producing 2000 bowls and by how much?
10. The cost of gas for welding 252 metal boxes is £3 11*s.* 0*d.* How much will it cost to weld 127 of these boxes?
11. A bronze is made up of copper, tin and zinc in the proportions 44 : 5 : 1. What weight of each is in 70 lb of the bronze?
12. In a certain brass the ratio of copper to zinc is 5 : 2. What weight of each is needed to make 28 lb of brass?

Averages

If a batch of trays were made in sheet metal, it would be extremely unlikely that each would weigh exactly the same as any other. If, however, the weight was important, this could be stated on the job order as ‘ Average weight to be 9 oz’.

If there were 12 trays the total weight would be:

$$12 \times 9 = 108 \text{ oz.}$$

If the total weight was given, the average weight would be found by:

$$\text{average weight} = \frac{\text{total weight}}{\text{number of trays}} = \frac{108}{12} = 9 \text{ oz.}$$

Thus the average weight is the *mean* or *middle* weight.

Notice that if the *average weight* and total number are known, the *total weight* can be found, or if the total and number are known the average can be found.

Example 1. *5 bags of rivets weigh respectively* 78, 75, 79, 77, 76 *lb. Calculate the average weight of the bags.*

$$\text{Total weight} = 78 + 75 + 79 + 77 + 76 = 385 \text{ lb}$$

$$\text{Average weight} = \frac{\text{total weight}}{\text{number of bags}} = \frac{385}{5} = \underline{77 \text{ lb}}$$

Example 2. *The average cost of material for making a sheet metal container is 9s. 6d. What would be the cost of making a gross of containers?*

$$\begin{aligned}\text{Total cost} &= \text{average cost} \times \text{quantity}\\ &= 9s.\ 6d. \times 144 = 1368s.\\ &= £68\ 8s.\end{aligned}$$

Example 3. *3 gangs of men work under a bonus system. 6 men earn £19 10s. in 4 days, 4 men earn £23 17s. in 5 days, 11 men earn £39 15s. in 3 days. Find the average bonus earned per man per day.*

Total bonus = £19 10*s*. + £23 17*s*. + £39 15*s*. = £83 2*s*.

This is the total of all the men's daily earnings.

6 men for 4 days earn 24 daily bonuses
4 men for 5 days earn 20 daily bonuses
11 men for 3 days earn 33 daily bonuses

$$\therefore\ \text{£83 2}s.\text{ is the total of } 24 + 20 + 33 = 77 \text{ daily bonuses}$$

$$\therefore\ \text{Average daily bonus per man} = \frac{\text{£83 2}s.}{77} = \underline{\text{£1 1}s.\ 7d.}$$

Percentages

A percentage is another method of stating a fraction. The fraction is expressed as a certain number of hundredths. By this method different fractions or ratios can be compared directly, as all the denominators are 100.

The whole, no matter what its value, is taken as 100 and the fractional part is stated as hundredths.

$$\tfrac{1}{2} = \tfrac{50}{100} = 50 \text{ per cent. or } 50\%$$
$$\tfrac{3}{4} = \tfrac{75}{100} = 75 \text{ per cent. or } 75\%$$
$$\tfrac{1}{5} = \tfrac{20}{100} = 20 \text{ per cent. or } 20\%$$

Note that the ' per cent.' sign % has the 1 and two zeros of 100.

Example 4. *The profit on a certain job is 40%. If it costs £57 10s. to make, what should the selling price be?*

$$\text{Profit} = 40\% = \tfrac{40}{100} \text{ of £57 10}s.$$
$$= \tfrac{2}{5} \times \text{£57 10}s. = \underline{\text{£23.}}$$
$$\therefore\ \text{Selling price} = \text{cost} + \text{profit} = \text{£57 10}s. + \text{£23}$$
$$= \underline{\text{£80 10}s.}$$

Example 5. *A finished part weighs 56 lb and the scrap weighs 12 lb. What percentage of the total metal used is scrap?*

$$\text{Total weight} = 56 + 12 = 68 \text{ lb and this is } 100\%$$
$$\tfrac{12}{68} \text{ is scrap}$$
$$\% = \tfrac{12}{68} \text{ of } 100 = \underline{17{\cdot}65\%.}$$

Using the proportion method:

$$100\% \text{ is } 68 \text{ lb}$$
$$x\% \text{ is } 12 \text{ lb}$$
$$x = 100 \times \tfrac{12}{68} = \underline{17{\cdot}65\%.}$$

EXAMPLE 6. *A solder is made up of 9 parts tin, 15 parts lead, 2 parts antimony by weight. What percentage of each is present?*

Total weight $= 9 + 15 + 2 = 26$ parts and this is 100%.

1 100% is 26
x% is 9
$x = 100 \times \frac{9}{26} = 34{\cdot}62\%$ tin.

2 100% is 26
x% is 15
$x = 100 \times \frac{15}{26} = 57{\cdot}69\%$ lead.

3 100% is 26
x% is 2
$x = 100 \times \frac{2}{26} = 7{\cdot}69\%$ antimony.

Or, since to change a fraction into a percentage we multiply by 100:

tin is $\frac{9}{26} = \frac{9}{26} \times 100 = 34{\cdot}62\%$
lead is $\frac{15}{26} = \frac{15}{26} \times 100 = 57{\cdot}69\%$
antimony is $\frac{2}{26} = \frac{2}{26} \times 100 = 7{\cdot}69\%$

EXAMPLE 7. *A weekly bonus of 40% is paid on a basic rate of £9. How much bonus is earned?*

$$40\% = \frac{40}{100} = \frac{2}{5}$$

$$\text{bonus} = \tfrac{2}{5} \text{ of } £9 = £3\tfrac{3}{5} = \underline{£3\ 12s.}$$

EXAMPLE 8. *From a sheet 6 ft long × 2 ft wide 8·3 sq. ft of metal is used. Calculate the percentage of scrap.*

Area of sheet $= 6 \times 2 = 12$ sq. ft
scrap $= 12 - 8{\cdot}3 = 3{\cdot}7$ sq. ft

$$\therefore \quad \%\ \text{scrap} = \frac{3{\cdot}7}{12} \times 100 = \frac{370}{12} = \underline{30{\cdot}83\%}$$

or, alternatively,

$$\%\ \text{scrap} = \frac{\text{total area} - \text{area used}}{\text{total area}} \times 100$$

$$= \frac{12 - 8{\cdot}3}{12} \times 100 = \underline{30{\cdot}83}\%\ \text{scrap}$$

Exercise 5

1. The lengths of five boxes are 8½ in., 9 in., 7⅝ in., 8¾ in. and 8¼ in. respectively. What is the average length?
2. To make an alloy the following metals are purchased:

48 lb at 2*s*. 8*d*. per lb
16 lb at 2*s*. 6*d*. per lb
8 lb at 3*s*. 0*d*. per lb.

What is the average price per lb of the resulting alloy?

3. In a class of 24 students one half receive full marks for working out a calculation. Of the remainder, one third receive 60% and two thirds receive 40% of the marks allocated. If the maximum number of marks obtainable is 20, what is the average mark of each student?

4. In a piecework pool of 30 men, 17 men earn £240 in a week, 9 men earn £100 in a week, whilst the remaining 4 are paid a flat rate of £14 each per week. What are the average weekly earnings of each man in the pool?
5. 9·38 sq. ft of metal are used in the making of a metal container. If the metal weighs 2·1 lb per sq. ft, calculate the weight of a gross of these containers.
6. The profit on a certain job is to be 72%. If the cost of manufacture is £15 8*s*., what should the selling price be?
7. An estimator quotes a selling price of a sheet metal component as £11 9*s*. 6*d*. If it costs £8 5*s*. 6*d*. to manufacture the components, what percentage of the selling price will be profit?
8. A sheet metal worker is required to cut a strip of metal ·75 in. wide. When measured with a micrometer the strip is found to be ·744 in. wide. Determine the percentage error.
9. A cistern holds 250 gallons. 4% is used, then 5% leaks away. If, now, 25% of the remainder is sold, how many gallons remain in the cistern?
10. From a sheet weighing 25 lb a motor car pressing is made and then 4·18 lb of metal are cut away as scrap. What percentage of the original sheet is used in the pressing, and what is the ratio of the scrap to the used metal?

Squares and Square Roots

When a number is multiplied by itself the product is called the *square of the number*.

For example:

$3 \times 3 = 9$, therefore 9 is the square of 3

$7 \times 7 = 49$, therefore 49 is the square of 7

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, therefore $\frac{1}{4}$ is the square of $\frac{1}{2}$

$0{\cdot}2 \times 0{\cdot}2 = {\cdot}04$, therefore ·04 is the square of ·2

Note that when a fraction is squared the value is decreased.

When a number, say 5, is to be squared it is written thus: 5^2. The small 2 above the figure is called an index or power and shows the number of fives to be multiplied.

$$5^2 = 5 \times 5 = 25.$$

The cube of a number is written thus: 5^3.

$$5^3 = 5 \times 5 \times 5 = 125.$$

$$0{\cdot}4^3 = 0{\cdot}4 \times 0{\cdot}4 \times 0{\cdot}4 = 0{\cdot}064.$$

To square a vulgar fraction it must be written thus:

$$\left(\frac{3}{4}\right)^2 \textbf{ not } \frac{3^2}{4}$$

$(\frac{3}{4})^2$ means; three quarters squared

$\frac{3^2}{4}$ means; three squared divided by 4.

$$(\tfrac{3}{4})^2 = \tfrac{9}{16}$$

$$\tfrac{3^2}{4} = \tfrac{9}{4} = 2\tfrac{1}{4}.$$

Square Roots

As the square of 2 is 4 so the square root of 4 is 2 and is written $\sqrt{4} = 2$; $\sqrt{\ }$ being the square root sign.

$$\sqrt{25} = 5. \qquad \sqrt{0{\cdot}09} = {\cdot}3.$$

$$\sqrt{36} = 6. \qquad \sqrt{{\cdot}16} = {\cdot}4.$$

A cube root if required is written $\sqrt[3]{\ }$.

$$\sqrt[3]{8} = 2.$$

If the square root of a vulgar fraction is to be found it must be written

$$\sqrt{\frac{9}{16}} \text{ not } \frac{\sqrt{9}}{16}$$

$$\sqrt{\frac{9}{16}} = \frac{3}{4}$$

$$\frac{\sqrt{9}}{16} = \frac{3}{16}.$$

Sometimes it is simpler to consider the top and bottom of a vulgar fraction separately:

$$\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}.$$

$$\sqrt{\frac{49}{64}} = \frac{\sqrt{49}}{\sqrt{64}} = \frac{7}{8}.$$

To Find the Square Root of a Number

Find the square root of 64516.

Put a frame around the number as shown.

Divide the number into pairs starting from the decimal point and working to the left.

By trial find the approximate square root of the first number or pair of numbers, in this example it is 2.

Place a 2 on the root line above the 6 and a 2 alongside the 6.

Multiply out the 2 × 2 and subtract from the 6.

	2	5	4 root.
2	$\overline{6}$	$\overline{45}$	$\overline{16}$
	4		
45	2	45	
	2	25	
504		20	16
		20	16
		..	..

Double the 2 in the root line and place this 4 beside the remainder, leaving space to its right for another number.

Bring down the next pair, 45.

By trial find a number which when put on the root line and beside the 4 and multiplied out will be just smaller than 245.

5 is suitable.

Multiply the 5 by the 45 and subtract from 245 leaving 20.

Double the root line number, $25 \times 2 = 50$.

Place this 50 alongside the 20 remainder, leaving space to the right.

Bring down the next pair, 16.

By trial 4 is the next root line number, place it beside the 50, multiply out 4×504 and subtract from 2016 leaving zero.

From the root line, $\sqrt{64516} = \underline{254}$.

If the number from which the square root is to be extracted has a decimal fraction the pairs are marked off to right and left of the decimal point.

EXAMPLE 1. *Find the value of* $\sqrt{762{\cdot}879}$.

Note a zero is placed after the 9 to make a pair, but the 7 is left alone.

$$\begin{array}{r|l}
 & 2\ \ 7\cdot 6\ \ 2\ \text{root} \\ \hline
2 & \overline{7}\ \overline{62}\cdot\overline{87}\ \overline{90} \\
 & 4 \\ \hline
47 & 3\ 62 \\
 & 3\ 29 \\ \hline
546 & \quad 33\ 87 \\
 & \quad 32\ 76 \\ \hline
5522 & \qquad 1\ 11\ 90 \\
 & \qquad 1\ 10\ 44 \\ \hline
 & \qquad .\ .1\ 46 \\ \hline
\end{array}$$

The remainder is left, as the extracted root is considered to be sufficiently accurate.

$\therefore \quad \sqrt{762{\cdot}879}$ is approx. 27·62.

EXAMPLE 2. *Extract the square root of* 0·002916.

Note the first figure after the point is zero.

$$\begin{array}{r|r}
 & \cdot\ 0\ \ 5\ \ 4 \\ \hline
05 & \cdot 00\ 29\ 16 \\
 & 25\ \\ \hline
 & 4\ 16 \\
104 & 4\ 16 \\ \hline
 & .\ .\ . \\ \hline
\end{array}$$

$\therefore \quad \sqrt{\cdot 002916} = \underline{\cdot 054}$.

Exercise 6

Find the value of:

1. (*a*) 25^2; (*b*) $2{\cdot}5^2$; (*c*) $0{\cdot}25^2$.
2. (*a*) $\left(\frac{7}{8}\right)^2$; (*b*) $\frac{7^2}{8}$.
3. (*a*) $\sqrt{64}$; (*b*) $\sqrt{121}$; (*c*) $\sqrt{81}$.
4. (*a*) $\sqrt{2116}$; (*b*) $\sqrt{45369}$; (*c*) $\sqrt{28{\cdot}09}$.
5. (*a*) $\sqrt{\frac{9}{64}}$; (*b*) $\sqrt{\frac{9}{16}}$; (*c*) $\sqrt{2\frac{1}{4}}$.

Indices and Logarithms

It has already been shown (p. 23), that the power or index of a number shows how many times a number is multiplied by itself:

$$r^1 = r$$

$$r^2 = r \times r$$

$$r^3 = r \times r \times r$$

now $r^2 \times r^2 = (r \times r) \times (r \times r) = r^4$

and $r^3 \times r^2 = (r \times r \times r) \times (r \times r) = r^5$

From the last two examples it is seen that to *multiply* numbers having indices, the indices are merely *added.*

Consider $r^4 \div r^2 = \dfrac{r \times r \times r \times r}{r \times r} = r^2$

and $r^{(4-2)} = r^2$

$r^3 \div r^2 = \dfrac{r \times r \times r}{r \times r} = r$

and $r^{3-2} = r$

so $r^5 \div r^3 = r^{(5-3)} = r^2$

These examples show that to *divide* numbers having indices, the indices are *subtracted.*

How much labour would be saved if all multiplication and division could be done as easily as this! It can be as easy as this if logarithms are used.

The numbers are first expressed as powers of 10 and then the multiplication and division is done merely by adding or subtracting the indices.

Logarithms

The small 2 above the 10 in 10^2 which we have called a power or index, can also be called a **logarithm**, or '**log**' for short.

$$100 = 10 \times 10 = 10^2$$

and the log of 100 is 2 with the base of 10.

Similarly, $10 = 10^1$ or log 1

$100 = 10^2$ or log 2

$1000 = 10^3$ or log 3

$10{,}000 = 10^4$ or log 4

Following the pattern,

$1 = 10^0$ or log 0.

Any other numbers, besides multiples of 10, can be expressed as logs to the base 10, but these numbers will have fractional indices or logs.

Thus, $100 = 10^2$ and log 100 is 2,

$1000 = 10^3$ and log 1000 is 3.

The log of 875 will be greater than log 100 and smaller than log 1000. It will be 2 plus a fraction and in fact, $875 = 10^{2\cdot9420}$, i.e., log 875 is 2·9420.

In this way, all common numbers can be expressed as powers, or logs, to the base 10 and these logs are obtained from log tables.

To multiply two numbers using log tables, the logs of the numbers are first obtained and the logs are then added. Again using the tables, the sum obtained is converted back from a log to a number. This number is the **product** of the two numbers to be multiplied.

EXAMPLE 1. *Multiply* 678 *by* 2351.

No.	Log	
678	2·8312	
2351	3·3713	
	6·2025	adding

The log 6·2025 as a number, is 1594000.

i.e., $678 \times 2351 = \underline{1{,}594{,}000}$.

Instead of a long-winded multiplication the process is done by simple addition, using logs.

The whole number part of a log (it may be 0) is called the **characteristic.** The fraction part is called the **mantissa.**

The characteristic part is found by inspection as that of 678 was found above.

$10 = 10^1$ and log 10 = 1,

$100 = 10^2$ and log 100 = 2.

Therefore all numbers between 10 and 100 will have a characteristic of 1.

$1000 = 10^3$ and log 1000 = 3.

Therefore all numbers between 100 and 1000 will have a characteristic of 2.

The rule is: the characteristic is one less than the number of figures representing the whole numbers.

364 has three whole figures, ∴ the characteristic is $3 - 1 = 2$
5789 has four whole figures, ∴ the characteristic is $4 - 1 = 3$
7 has one whole figure, ∴ the characteristic is $1 - 1 = 0$

26·43 has two whole figures, ∴ the characteristic is 2 − 1 = 1
378·4 has three whole figures, ∴ the characteristic is 3 − 1 = 2

(Fractions without a whole number will obviously have negative characteristics, but these are outside the scope of this book.)

The mantissa is found from log tables.

Reading Log Tables

To find the log of 4658.

The characteristic is 4 − 1 = 3, and is written down 3·

The first two figures (46) are found in the left-hand vertical column.

Read along horizontally from the 46 to reach the column headed by the next figure (5). This will give the number ·6675 in the small box.

So far, we have the mantissa of 465 = ·6675.

Carry along to the columns headed ' mean difference ' to the number underneath the last figure 8,. This gives the figure 7. Add this to 6675.

The mantissa of 4658 = 6675 + 7 = 6682, and log 4658 is 3·6682.

(Some log tables have no ' mean difference ' heading but simply numbers 1 to 9, in a separate heading.)

The following example summarises the procedure.

EXAMPLE 2. *Find the log of* 465·8.

(a) The characteristic = 3 − 1 = 2.

(b) The mantissa. The decimal point is ignored and 4658 is looked up in the tables, this will be, as before, ·6682.

∴ Log 465·8 is $\underline{2{\cdot}6682}$.

For the same group of figures, only the characteristic changes with the position of the decimal point, the mantissa remains the same no matter where the decimal point is.

Thus, log 4658 = 3·6682
log 465·8 = 2·6682
log 46·58 = 1·6682
log 4·658 = 0·6682
log 465800 = 5·6682

Summary

A log is an index to the base 10.

The whole number part is called the characteristic, and is one less than the number of figures in the whole number.

The fractional part is called the mantissa and it is found from log tables.

The mantissa is independent of the position of the decimal point for any set of figures.

To multiply numbers, change them to logs and add the logs. Convert the resultant log back into a number.

To divide, subtract the logs, as shown in later examples.

Antilogarithms

Log tables can be used to find the log of a number, or to find the number from a log by reversing the process. Antilog tables are designed to make this second part easier. By using antilog tables in the same way as log tables the number can be found from a log.

Only the mantissa is looked up in the antilog tables.

Always make sure you are using the correct table for the process you want to carry out. Log tables show all whole numbers, antilogs are shown as decimal fractions.

EXAMPLE 3. *Using logs, find the value of* 4682 × 7·98.

	No.	Log	Antilog
	4682	3·6704	
	7·98	0·9020	
adding		4·5724	3736 (with 5 whole numbers before the decimal point)
rough check			
4700 × 8 = 37600			

∴ 4682 × 7·98 = 37360.

The characteristic shows there are 4 + 1 whole numbers before the decimal point. This is the reverse of the way in which the characteristic is found, when a number is changed to a log.

Note that a rough check must be made to avoid errors in placing the decimal point.

EXAMPLE 4. *Evaluate* $\frac{379{\cdot}2 \times 6{\cdot}673}{85{\cdot}44 \times 2{\cdot}41}$, *using logs.*

rough check

$$\frac{\overset{19}{\cancel{3}\cancel{8}\cancel{0}} \times 7}{9\cancel{0} \times \cancel{2}} \bumpeq \frac{133}{9} \bumpeq 13$$

	No.	Log	Antilog	
	379·2	2·5788		
	6·673	0·8243		
	Adding,	3·4031		. . . (1)
	85·44	1·9317		
	2·41	0·3820		
	Adding,	2·3137		. . . (2)
	subtracting (2) from (1)	1·0894	12·28	

$$\therefore \frac{379{\cdot}2 \times 6{\cdot}673}{85{\cdot}44 \times 2{\cdot}41} = \underline{12{\cdot}28}$$

EXAMPLE 5. *The horse power transmitted by a belt driving a machine is given by* $H\frac{2\pi NT}{33000}$.

Find the horse power transmitted when N *is* 500 *and* T *is* 370.

$$H = \frac{2\pi NT}{33000}$$

rough check:

$$\frac{2 \times \cancel{3} \times 5\cancel{00} \times 37\cancel{0}}{\underset{11}{\cancel{3}\cancel{3}\cancel{000}}} \simeq 37$$

$$= \frac{2 \times 3{\cdot}14 \times 500 \times 370}{33000}$$

$$= \frac{10 \times 3{\cdot}14 \times 37}{33}$$

$$= \frac{31{\cdot}4 \times 37}{33}$$

No.	Log	Antilog
31·4	1·4969	
37	1·5682	
Adding,	3·0651	
33	1·5185	
Subtracting,	1·5466	35·21

∴ *Horse power transmitted* = 35·21.

Note.—The working should be simplified by cancellation and mental arithmetic whenever possible, before using logs.

Powers and Roots. To find by Logs

$$r^2 = r \times r, \text{ i.e., } \log r^2 = \log r + \log r$$
$$= 2 \log r$$
$$r^3 = r \times r \times r \text{ i.e., } \log r^3 = \log r + \log r + \log r$$
$$= 3 \log r.$$

∴ *To raise a number to a power multiply the log of the number by the power.*

$$\log \text{ of } r^2 = 2 \log r$$
$$\sqrt{r^2} = r$$
$$\text{and } \log \text{ of } \sqrt{r^2} = \frac{2 \log r}{2} = \log r$$
$$\log \text{ of } r^3 = 3 \log r$$
$$\sqrt[3]{r^3} = r$$
$$\text{and } \log \text{ of } \sqrt[3]{r^3} = \frac{3 \log r}{3} = \log r$$

∴ *To find the root of a number divide the log of the number by the required root.*

EXAMPLE 6. *Find the value of* $\sqrt{625}$.

No.	Log	Antilog
625	2·7959	
Divide by 2	1·3979	25

$$\therefore\ \sqrt{625} = \underline{25.}$$

EXAMPLE 7. *Find the value of* $\sqrt[3]{152{\cdot}7}$.

No.	Log	Antilog
152·7	2·1838	
Divide by 3	0·7279	5·344.

$$\therefore\ \sqrt[3]{152{\cdot}7} = \underline{5{\cdot}344}$$

EXAMPLE 8. *Find the value of* $465{\cdot}3^2$.

No.	Log	Antilog
465·3	2·6678	
Multiply by 2	5·3356	2166.

$$\therefore\ 465{\cdot}3^2 = \underline{216{,}600.}$$

EXAMPLE 9. *Find the value of* $60{\cdot}07^3$.

No.	Log	Antilog
60·07	1·7787	
	× 3	
	5·3361	2168

$$\therefore\ 60{\cdot}07^3 = \underline{216{,}800.}$$

EXAMPLE 10. *Evaluate:* $\dfrac{3{\cdot}072^3}{4{\cdot}61^2}$.

No.	Log	Antilog	
3·072	0·4873		
	× 3		
	1·4619		. . . (1)
4·61	0·6637		
	× 2		
	1·3274		. . . (2)
Subtract (2) *from* (1)	0·1345	1·363	

$$\therefore\ \frac{3{\cdot}072^3}{4{\cdot}61^2} = \underline{1{\cdot}363.}$$

As numbers are multiplied by adding their logs, they cannot be *added* by the use of logs. The results of multiplication must be found first. This is shown in the following calculation.

EXAMPLE 11. *Evaluate* $A = \sqrt{(B^2 + C^2)}$ *when* B = 10·48 *and* C = 8·75.

$$A = \sqrt{(10{\cdot}48^2 + 8{\cdot}75^2)}$$

No.	Log	Antilog	
10·48	1·0204 × 2		
	2·0408	109·8	
8·75	0·9420 × 2		
	1·8840	76·56	
		186·36	adding antilogs

No.	Log	Antilog
186·36	2·2704	
Divide by 2	1·1352	13·66

∴ A = 13·66.

Note—The values of B^2 and C^2 are added *not* the logs.

It has already been shown, p. 13, that many decimals are indeterminate. In the same way, many logs are approximations. If, then, the problem is one that can be solved by easy arithmetic, logs should not be used.

Exercise 7

1. Find the logs of:
   (*a*) 72, 7·2, 72000.
   (*b*) 10, 100, 1.
   (*c*) 7·008, 20050, 900·7.
   (*d*) 80·07, 52070, 100·6.
2. Find the numbers corresponding to:
   (*a*) log 1·6944, log 0·6944, log 3·6944.
   (*b*) log 0·0030, log 3·0000, log 1·3000.
   (*c*) log 2·6079, log 0·0485, log 0·4853.
   (*d*) log 5·8261, log 1·9890, log 0·8000.
3. Using logs find the value of:
   (*a*) 30·56 ÷ 4·105.
   (*b*) 23·51 ÷ 6·78.
   (*c*) 407·9 ÷ 14·56.
   (*d*) 4·127 × 3·142.
   (*e*) 31·7 × 9200.
   (*f*) 415·1 × 1·6.
   (*g*) 8·754 × 38·61.
   (*h*) 412300 ÷ 78·54.
   (*i*) 13·87 × 1·729 × 32·27 × 1·004.

4. Evaluate:

(*a*) $\dfrac{19\cdot25 \times 66\cdot71}{72\cdot13}$

(*b*) $\dfrac{64\cdot4 \times 418}{100\cdot25}$

(*c*) $\dfrac{90\cdot4 \times 3\cdot14 \times 500}{4\cdot12 \times 3306}$

(*d*) $\dfrac{7\cdot416 \times 365}{63\cdot5}$

(*e*) $\dfrac{61\cdot03 \times 5\cdot5}{39\cdot4 \times 1\cdot025}$

5. Evaluate by the use of logs:
(*a*) $\sqrt{1257}$, (*b*) $\sqrt{674\cdot3}$, (*c*) $\sqrt{5\cdot684}$, (*d*) $\sqrt{4\cdot599}$, (*e*) $\sqrt{269\cdot8}$,
(*f*) $\sqrt[3]{2}$, (*g*) $\sqrt[3]{1728}$, (*h*) $\sqrt[3]{100}$, (*i*) $\sqrt[3]{5149}$.

6. Solve:
(*a*) $39\cdot54^2$, (*b*) $1\cdot386^3$, (*c*) $1\cdot063^2$. (*d*) $35\cdot89^2$, (*e*) $16\cdot15^2$,
(*f*) 125^3.

7. Solve:

(*a*) $\sqrt{\dfrac{480}{70\cdot9}}$ (*b*) $\left(\dfrac{154\cdot30^2}{132\cdot7}\right)$

(*c*) $\sqrt[3]{\dfrac{289\cdot1}{37\cdot64}}$ (*d*) $\sqrt{\dfrac{16\cdot5 \times 11\cdot96}{2\cdot47 \times 13\cdot45}}$

(*e*) $\sqrt{\dfrac{6\cdot284 \times 1250}{981}}$ (*f*) $\sqrt[3]{\dfrac{3975}{1728}}$

8. Solve:

(*a*) $\sqrt{(2\cdot116^2 + 4\cdot272^2)}$ (*c*) $\sqrt{(1\cdot95^2 + 1\cdot3^2)}$

(*b*) $\sqrt{(400^2 + 1220^2)}$ (*d*) $\sqrt{(10^2 - 3\cdot75^2)}$

9. The radius of a sphere of volume V is given by the formula, $r = \sqrt[3]{\dfrac{3V}{4\pi}}$. Find the radius of a sphere of volume 600 cu. ft.

10. The surface area of a sphere is given by, $S = 4\pi r^2$, when r is the radius of the sphere. Find the surface area of a sphere of radius 15·8 in.

11. The volume of a sphere is found from $V = \dfrac{4}{3}\pi r^3$. Find the volume of a sphere of radius 2·25 in.

12. From the formula, $d = \sqrt{\dfrac{4a}{\pi}}$, find the value of d when $a = 28\cdot61$ and $\pi = 3\cdot14$.

13. From the formula $V = 10\cdot47(R^2 + r^2 + Rr)$, find the value of V when $R = 5$ and $r = 4$.

14. Solve $\sqrt{[8\cdot7(8\cdot7 - 7\cdot3)(8\cdot7 - 4\cdot7)(8\cdot7 - 5\cdot4)]}$.

15. From the formula $f = \dfrac{v^2}{r} \times \dfrac{L}{R}$ find the value of f when $v = 18\cdot33$, $r = 1\cdot25$, $L = 11\cdot5$ and $R = 15$.

16. Using logs find the value of W from the formula: $W = \dfrac{Fgt}{r^2p}$ when $F = 205\cdot9$, $g = 32\cdot2$, $t = 12$, $r = 64\cdot4$, $p = 15$.

17. Find the value of F from the formula: $F = \dfrac{W}{g}v^2r$ when $W = 50$, $g = 32\cdot2$, $v = 37\cdot7$ and $r = 1\cdot5$.

18. From the formula $v = \sqrt{\frac{K \times 2g}{W}}$ find the value of v when $K = 56000$, $g = 32{\cdot}2$ and $W = 5200$.

19. Using the formula $d = \frac{WL^3}{PKb^3}$ find the value of d when $W = 150$, $L = 216$, $K = 64$, $P = 700$, $b = 16$.

20. The horsepower developed by an engine is found from the formula $H = \frac{PLAN}{33000}$. Calculate the horsepower developed when $P = 40$, $L = 1{\cdot}5$, $A = 75{\cdot}4$, $N = 250$.

Transposition of Formulae

Many practical calculations involve the use of a formula (plural formulae), which may be obtained from books of reference or may be proved, though none of these proofs is asked for in the City and Guilds of London Institute course on sheet metal work, and therefore in many cases these formulae are put forward as propositions.

Suppose it is required to calculate the length of metal to make a cylinder, knowing its diameter. Then the circumference, or the distance round the circle, would be obtained by using the formula $C = \pi d$,

where C = circumference of circle,
$\pi = 3{\cdot}14$ (*always*),
d = diameter of circle.

(*Note* that in formulae the × sign is omitted.)

However, should a piece of metal of known length be rolled into a cylinder, the diameter may be predetermined by rearranging the formula. This process of rearrangement is known as **transposition.**

In any formula consider the equals sign (=) to be a balance, and any operation carried out on one side must be done on the other to maintain that balance. Hence, one side of a formula can be subtracted from, multiplied or divided by any number, providing exactly the same thing is done to the other side, thus keeping the whole equation in balance.

EXAMPLE 1. *Find the diameter* d *from the formula* $C = \pi d$.

Dividing both sides by π $\quad \frac{C}{\pi} = \frac{\pi d}{\pi}$

Cancelling $\quad \frac{C}{\pi} = d$

Rewriting $\quad d = \frac{C}{\pi}$

Example 2. *The thickness of a ring* t *is given by* $t = R - r$. (a) *Find* R *if* t *and* r *are known.* (b) *Find* r, *if* t *and* R *are known.*

(*a*) $t = R - r$

Adding r to both sides, $t + r = R - r + r$

Cancelling $t + r = R$

Rewriting $\underline{R = t + r}$

(*b*) $t = R - r$

Adding r to both sides, $t + r = R - r + r$

Cancelling $t + r = R$

Subtracting t from both sides,

$$t + r - t = R - t$$

Cancelling $\underline{r = R - t}$

Example 3. *The area of a circle is given by* $A = \pi r^2$. *Transpose this formula to give* r *in terms of the others.*

$$A = \pi r^2$$

Dividing by π $\frac{A}{\pi} = \frac{\pi r^2}{\pi}$

$$\therefore \frac{A}{\pi} =$$

Taking the square root of both sides $\sqrt{\frac{A}{\pi}} = \sqrt{r^2} = r$

Rewriting $\underline{r = \sqrt{\frac{A}{\pi}}}$

Example 4. *The depth of a tank is given by: depth* $d = \frac{G \times 277}{LB}$. *Make* (*i*) G, (*ii*) L, (*iii*) B *the subject of the formula.*

(i) $d = \frac{G \times 277}{LB}$

$$dLB = \frac{G \times 277}{LB} \times LB$$

$$\frac{dLB}{277} = \frac{G \times \cancel{277}}{\cancel{277}}$$

$$\underline{G = \frac{dLB}{277}}$$

(ii)
$$d = \frac{G \times 277}{LB}$$
$$dL = \frac{G \times 277}{LB} \times L$$
$$dL = \frac{G \times 277}{B}$$
$$\frac{dL}{d} = \frac{G \times 277}{Bd}$$
$$L = \frac{G \times 277}{Bd}.$$

(iii)
$$d = \frac{G \times 277}{LB}$$
$$dB = \frac{G \times 277}{LB} \times B$$
$$dB = \frac{G \times 277}{L}$$
$$\frac{dB}{d} = \frac{G \times 277}{dL}$$
$$B = \frac{G \times 277}{dL}$$

EXAMPLE 5. *The area of a trapezium is found from:* $A = \frac{a+b}{2} \times h$. *Transpose to make* (i) a, (ii) b, (iii) h *the subject of the formula.*

(i)
$$A = \frac{a+b}{2} \times h$$
$$\frac{A}{h} = \frac{a+b}{2} \times \frac{h}{h}$$

Notice that $a + b$ cannot be separated until they are isolated.

$$\frac{A}{h} = \frac{a+b}{2}$$
$$\frac{2A}{h} = \frac{a+b}{\cancel{2}} \times \cancel{2}$$
$$\frac{2A}{h} = a + b$$
$$\frac{2A}{h} - b = a + b - b$$
$$\therefore \quad a = \frac{2A}{h} - b.$$

(ii) $$A = \frac{a+b}{2} \times h$$

$$\frac{A}{h} = \frac{a+b}{2}$$

$$\frac{2A}{h} = a + b$$

$$\underline{b = \frac{2A}{h} - a}$$

(iii) $$A = \frac{a+b}{2} \times h$$

$$2A = (a+b)h$$

$$\frac{2A}{a+b} = h$$

$$\underline{h = \frac{2A}{a+b}}$$

EXAMPLE 6. *If* $d = \frac{3Wl^2}{2T}$, *change the subject to give an expression for* l.

$$d = \frac{3Wl^2}{2T}$$

$$\therefore \quad 2dT = 3Wl^2$$

$$\frac{2dT}{3W} = l^2$$

Taking the square root of both sides,

$$\sqrt{\frac{2dT}{3W}} = l$$

Note.—The expression $\frac{2dT}{3W}$ must first be found before the square root is taken.

EXAMPLE 7. *The capacity in gallons* (G) *of a square sectional water tank is obtained from the formula* $G = 6{\cdot}24\,L^2D$. *Where* L = *length of the sides in feet,* D = *depth of the tank in feet, find the length of the sides when the capacity is* 5 *gallons and the depth is* 6 *in.*

$$\text{Now } G = 6{\cdot}24\,L^2D$$

$$\frac{G}{6{\cdot}24} = L^2D$$

$$\frac{G}{6{\cdot}24D} = L^2$$

$$\sqrt{\frac{G}{6{\cdot}24D}} = L$$

Substituting values gives $L = \sqrt{\frac{5}{6{\cdot}24 \times {\cdot}5}}$ Note, 6 in. = ·5 ft.

$$L = \sqrt{1{\cdot}603}$$

$$= \underline{1{\cdot}266 \text{ ft.}}$$

Exercise 8

1. Rearrange the following formulae as shown:
   (i) $a = 4 + bh$; make b then h the subject.
   (ii) $a = bh$; make b then h the subject.
   (iii) $v = \frac{1}{3}abc$; make a the subject.

2. Rearrange the following formulae to make t the subject:
   (i) $S = 2t$, (ii) $S = \frac{1}{2}t$, (iii) $S = vt$, (iv) $S = 2v + t$.

3. Rearrange the following formulae to make d the subject:
   (i) $S = ad - 2$, (ii) $a + 2 = 4d$, (iii) $S = 3d + t$.

4. Rearrange the following formulae to make t the subject:
   (i) $S = \frac{vt}{2}$, (ii) $E = \frac{lt}{d}$, (iii) $h = \frac{4tp}{9}$, (iv) $R = \frac{6at}{5b}$.

5. Make t the subject of the formula: $S = \frac{1}{2}gt^2$.

6. Make r the subject of the formula: $V = \pi r^2 h$.

7. Make d the subject of the formula: $T = \frac{pld^2}{D}$.

8. Make V the subject of the formula: $P = \frac{V^2}{R}$.

9. The approximate pressure required to shear a plate is given by: $P = Ltf$, where P is the shear pressure, L is the length sheared, t is the plate thickness, f is the shear strength of the plate.

   Transpose this formula to make L, t, f the subject. Then calculate the length of plate which can be sheared when $P = 40$ tons, $t = 0{\cdot}065$ in. and $f = 20$ tons/in^2.

10. The approximate bending force in Vee dies per foot length of metal is given by: $P = \frac{428t^2}{V}$ tons, where t is the stock thickness, and V is the width of die opening.

    Make V, t the subject of the formula and calculate the stock thickness when $P = 9{\cdot}56$ tons and $V = 0{\cdot}5$ in.

11. The diameter of rivet (d) to be used on metal of a certain thickness is given by: $d = \sqrt{t} \times 1{\cdot}2$. Calculate the sheet thickness t when the rivet dia. is $0{\cdot}25$ in.

12. If the area of a circle of radius r is given by $A = \pi r^2$, calculate the radius of a circle of area $37{\cdot}4$ square in.

Mensuration of Plane Figures

A quadrilateral is a plane figure bounded by four straight lines, such as the square, rectangle, rhombus and rhomboid.

Square and Rectangle

By drawing a rectangle 3 in. by 2 in. and dividing it into strips of 1 in. length and breadth, it will be found that the area of the rectangle is now made up of six small squares (Fig. 9). But the length multiplied by the breadth is 3 in. × 2 in. = 6 sq. in. Therefore the area of a rectangle = length × breadth, and similarly the area of a square = length × length = length^2.

Notice that to obtain the term 6 sq. in. the numbers have been multiplied ($3 \times 2 = 6$) and also the values of the units (inches × inches = square inches). Inches and feet, or feet and yards, cannot be multiplied together without first expressing them in similar units.

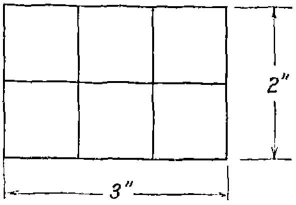


FIG. 9

EXAMPLE 1. *Calculate the weight of a rectangular sheet 4 ft 3 in. long by 2 ft 6 in. wide if the metal weighs 2·7 lb per sq. ft.*

$$
\begin{aligned}
4 \text{ ft } 3 \text{ in.} &= 4{\cdot}25 \text{ ft}\\
2 \text{ ft } 6 \text{ in.} &= 2{\cdot}5 \text{ ft}\\
\therefore \quad \text{Area} &= 4{\cdot}25 \times 2{\cdot}5 = 10{\cdot}625 \text{ sq. ft.}\\
\text{Weight} &= 10{\cdot}625 \times 2{\cdot}7 = 28{\cdot}6875 \text{ lb}\\
&= \underline{28{\cdot}69 \text{ lb}} \text{ to four significant figures.}
\end{aligned}
$$

EXAMPLE 2. *A square sheet of metal has an area of 30¼ sq. in. Determine the length of its sides.*

$$
\begin{aligned}
\text{Now, length}^2 &= \text{area}\\
\therefore \quad l^2 &= 30\tfrac{1}{4} \text{ sq. in.}\\
l &= \sqrt{30\tfrac{1}{4}} \text{ in.}\\
&= \sqrt{\frac{121}{4}} \text{ in.} = \frac{\sqrt{121}}{\sqrt{4}} \text{ in.}\\
\text{length} &= \tfrac{11}{2} = \underline{5\tfrac{1}{2} \text{ in.}}
\end{aligned}
$$

EXAMPLE 3. *A rectangular tank with an open top measures 3 ft × 2½ ft × 4 ft deep, and is to be made of metal weighing 3½ lb/sq. ft. Calculate the weight of the tank and also the percentage of scrap metal if 60 sq. ft of metal is used in making the tank.*

If the tank sides are considered to be opened out (Fig. 10),

$$
\begin{aligned}
\text{the total length} &= 3 + 2\tfrac{1}{2} + 3 + 2\tfrac{1}{2} = 11 \text{ ft,}\\
\text{area of sides} &= 11 \times 4 = 44 \text{ sq. ft,}\\
\text{area of bottom} &= 3 \times 2\tfrac{1}{2} = 7\tfrac{1}{2} \text{ sq. ft,}\\
\text{total area} &= 44 \times 7\tfrac{1}{2} = 51\tfrac{1}{2} \text{ sq. ft.}
\end{aligned}
$$

Now, the weight of the tank is

$$51\tfrac{1}{2} \times 3\tfrac{1}{2} = \underline{180\tfrac{1}{4} \text{ lb,}}$$

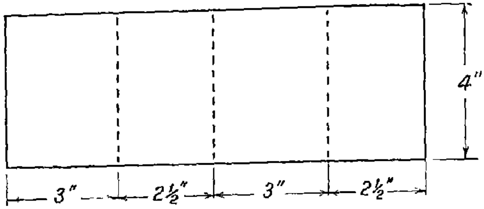


FIG. 10

and the percentage of scrap metal (P)

$$= \frac{\text{area used} - \text{area in tank}}{\text{area used}} \times \frac{100}{1}$$

$$\therefore \quad P = \frac{60 - 51\frac{1}{2}}{60} \times \frac{100}{1}$$

$$= \frac{8\frac{1}{2}}{\underset{3}{\cancel{60}}} \times \frac{\overset{5}{\cancel{100}}}{1} = \frac{42\frac{1}{2}}{3}$$

$$= \frac{85}{2} \times \frac{1}{3} = \frac{85}{6}$$

$$\text{Scrap} = \underline{14\tfrac{1}{6}\%} \text{ or } \underline{14{\cdot}166\%}.$$

A **parallelogram** is a four-sided figure with opposite sides equal and parallel. The angles are not right angles, but if all the sides are of equal length the figure is known as a **rhombus**.

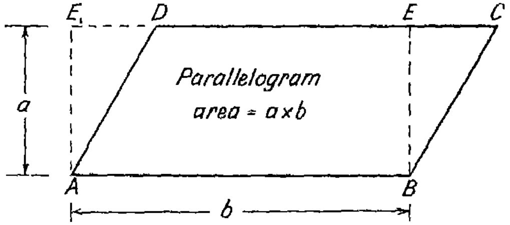


FIG. 11

Consider the parallelogram ABCD (Fig. 11). By removing the triangle CBE and transferring it to the other side, it is evident that the parallelogram can be made into a rectangle. Hence the area of the parallelogram is the product of the base and altitude, i.e.,

$$\text{area} = a \times b.$$

EXAMPLE 4. *A punch, in the form of a parallelogram, is to punch out blanks of $4\frac{3}{8}$ sq. in. If the distance between two of its sides is $1\frac{3}{4}$ in., determine the length of the base.*

$$\text{Area} = \text{base} \times \text{altitude}$$

$$\therefore \quad 4\tfrac{3}{8} = \text{base} \times 1\tfrac{3}{4}$$

from which,

$$\text{base} = \frac{4\frac{3}{8}}{1\frac{3}{4}} = \underline{2\tfrac{1}{2}\text{ in.}}$$

A **trapezium** (Fig. 12) may be treated in a similar manner, except that the triangle which is removed from one side is inverted when placing it on the other. In this way the altitude remains the same, but the base is the average between the top and bottom. Hence the area of a trapezium is the product of the altitude and one half the sum of the parallel sides,

i.e., $\quad \text{area} = a \times \dfrac{b + c}{2}.$

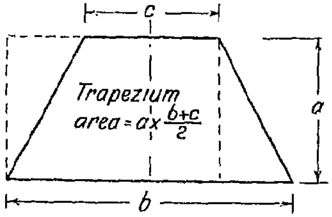


FIG. 12

EXAMPLE 5. *A fume extraction hood is made of four similar plates, each in the form of a trapezium. Calculate the weight of the hood if the base is 4 ft 6 in., the top is 9 in. and vertical height 3 ft. Assume the metal to weigh 5 lb/sq. ft.*

$$\begin{aligned}
\text{Area of one plate} &= 3 \times \frac{4\frac{1}{2} + \frac{3}{4}}{2} \\
&= 3 \times 2\tfrac{5}{8} = 7\tfrac{7}{8} \text{ sq. ft.} \\
\text{Area of hood} &= 7\tfrac{7}{8} \times 4 \\
\text{Weight of hood} &= 7\tfrac{7}{8} \times 4 \times 5 \\
&= \underline{157{\cdot}5 \text{ lb.}}
\end{aligned}$$

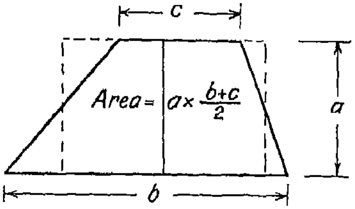


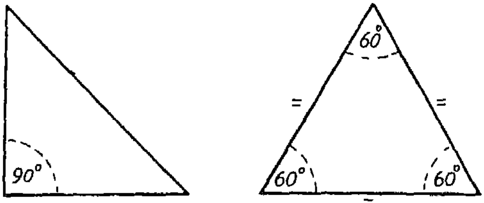
FIG. 13

The formula for the trapezium still holds when the figure is not symmetrical about its central axis, and this is shown in Fig. 13, where the average length of the base is obtained by the reversal of two small triangles.

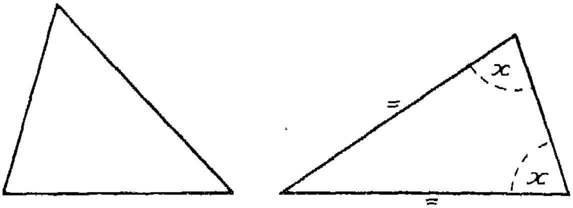
The Triangle

Triangles are classified as follows (Fig. 14):

(*a*) **Right angled**, when any one angle is a right angle, i.e., 90°.
(*b*) **Equilateral**, where all the sides and angles are equal.
(*c*) **Scalene**, when all sides and angles are unequal.
(*d*) **Isosceles**, when any two sides and angles are equal.



Right angle *Equilateral*



Scalene *Isosceles*

Fig. 14

The sum of the interior angles of any triangle is 180°.

As a triangle has three sides it has three angular points, and any one of these may be called a **vertex**. The opposite side to a vertex is called the **base**. In a right angled triangle the longest side is opposite the right angle and is known as the **hypotenuse**.

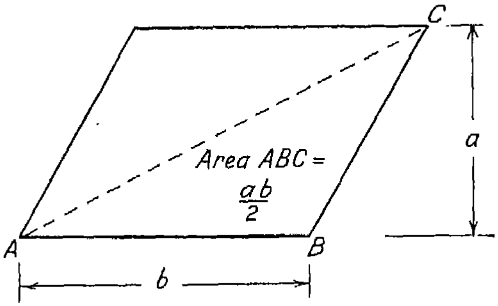


Fig. 15

Fig. 15 shows that any parallelogram cut by a diagonal is divided into two equal triangles, and, since the area of a parallelogram is the product of the base and the altitude, then the area of a triangle is one half of this amount, i.e., $\frac{a \times b}{2}$, or

the area of a triangle is half the base × vertical height.

EXAMPLE 6. *100 triangular blanks of 10 in. base and 9 in. vertical height are cut from a sheet 8 ft. × 4 ft. Calculate the area of the blanks and the percentage of the sheet used in forming the blanks.*

$$\text{Area of one triangle} = \frac{\text{base} \times \text{height}}{2}$$

$$= \frac{10 \times 9}{2} = 45 \text{ sq. in.}$$

$$\text{Area of metal used} = 45 \times 100 = 4500 \text{ sq. in.}$$

$$= \frac{4500}{144} = 31{\cdot}25 \text{ sq. ft.}$$ (To change sq. in. to sq. ft. divide by 12 × 12 = 144.)

$$\text{Area of sheet} = 8 \times 4 = 32 \text{ sq. ft.}$$

$$\text{Percentage used} = \frac{31{\cdot}25}{32} \times 100 = \underline{97{\cdot}66\%}.$$

The Theorem of Pythagoras

An important application of squares and square roots appears in the right angled triangle.

Consider the triangle ABC (Fig. 16), where it will be found that

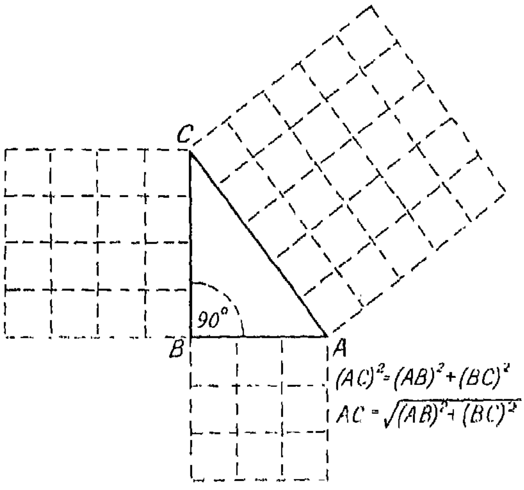


FIG. 16

if side AB = 3 in., BC = 4 in. and the angle ABC is a right angle, then the hypotenuse AC is 5 in. If squares are drawn on each of the sides, their areas are

$$3^2 = 9 \text{ sq. in.}$$

$$4^2 = 16 \text{ sq. in.}$$

$$5^2 = 25 \text{ sq. in.}$$

Now, $9 + 16 = 25$ and the result suggests that:

The square on the hypotenuse is equal to the sum of the squares on the other two sides,

i.e., $$AC^2 = AB^2 + BC^2$$

This is only true for right angled triangles and is known as the **theorem of Pythagoras.**

If, instead of erecting squares on the sides, circles with diameters AB, BC and AC were drawn, the relationship would still be true (Fig. 17).

Hence the area of a 5 in. diameter circle is equal to the sum of the areas of a 3 in. and a 4 in. diameter circle. This method is used extensively in the design of sheet metal ductwork where the diameter of a main pipe is required to take up the same area as two or more branch pipes.

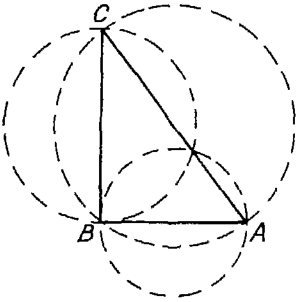


FIG. 17

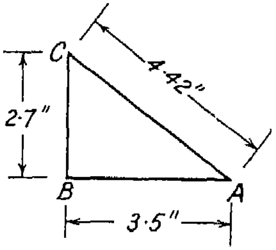


FIG. 18

EXAMPLE 7. *Determine graphically the diameter of a main fume extraction pipe to take up the flow from two branches* 3·5 *in. diameter and* 2·7 *in. diameter respectively.*

Draw AB = 3·5 and BC = 2·7 in. at right angles to AB. The length AC will give the diameter for the main, i.e., 4·42 in. (Fig. 18).

EXAMPLE 8. *Determine the minimum diameter of a main duct pipe to take up four branches* 2, 2·5, 2·75 *and* 3·25 *in. diameter* (Fig. 19).

This is merely an extension of the previous example

(*a*) Draw BC at right angles to AB
(*b*) Draw CD at right angles to AC
(*c*) Draw DE at right angles to AD.

The length AE will be the diameter of the main duct = 5·327 in. dia.

This method of graphical determination may only be used when all the pipes in the system are either square or round. In the event of mixed pipes being used the full calculations must be carried out.

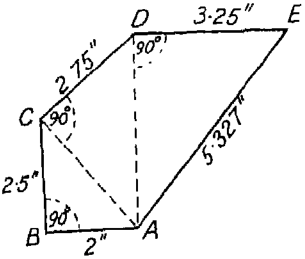


FIG. 19

Conic Radii

The radius of a pattern for a right cone may be calculated when the vertical height and base diameter are given. A right angled triangle in which one side is equal to the vertical height and another equal to the half diameter of the base should be used to calculate the length of the hypotenuse, which will represent the slant side of the cone.

EXAMPLE 9. *The vertical height of a right cone is* 11 *in. and the base diameter is* 6 *in. Calculate the pattern radius* (Fig. 20).

AD = 3 in. and CD = 11 in.

Now, $AC^2 = CD^2 + AD^2$

$AC^2 = 11^2 + 3^2 = 121 + 9 = 130$

$\therefore \quad AC = \sqrt{130} = 11{\cdot}4$ in.

$\therefore$ pattern radius = 11·4 in.

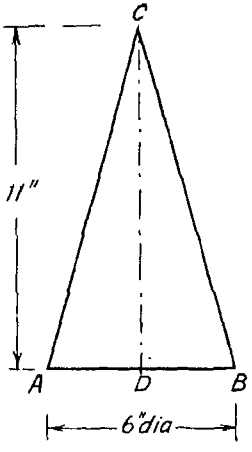


FIG. 20

In practice, the pattern radii for a right conic frustum are more often required, and the dimensions usually given are the top and bottom diameters and the vertical height of the frustum.

Fig. 21 represents a right conic frustum ABCD with diameters AB and CD. The line DG is the vertical height of the frustum, while FE is the full height of the cone. Lengths AF and DF are the required radii for the pattern.

Now, triangles AGD and AEF are similar and all their sides are directly proportional to each other.

By the Theorem of Pythagoras.

$$AD^2 = AG^2 + GD^2$$

$$AD = \sqrt{(AG^2 + GD^2)}$$

and $$AF = \frac{AE}{AG}\sqrt{(AG^2 + GD^2)} = \textbf{major radius}$$

but $$DF = AF - AD$$

$$= \frac{AE}{AG}\sqrt{(AG^2 + GD^2)} - \sqrt{(AG^2 + GD^2)} = \textbf{minor radius}$$

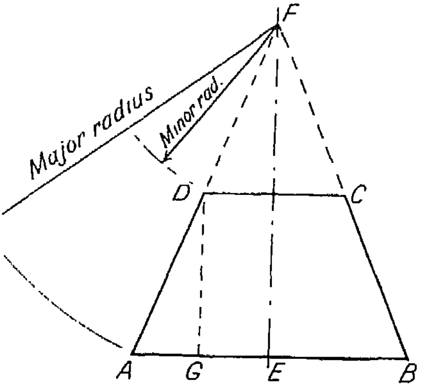


FIG. 21

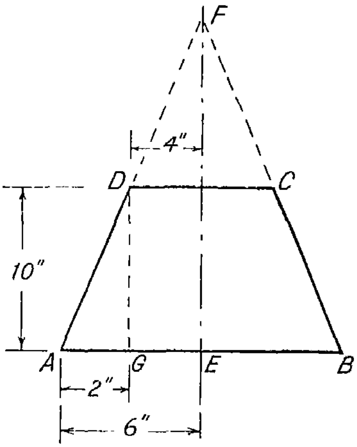


FIG. 22

EXAMPLE 10. *A right cone frustum is* 8 *in. diameter at the top,* 12 *in. diameter at the bottom and has a vertical height of* 10 *in. Calculate the major and minor pattern radii* (Fig. 22).

$$\text{Now major radius AF} = \frac{AE}{AG}\sqrt{(AG^2 + GD^2)}$$

$$= \frac{6}{2}\sqrt{(2^2 + 10^2)}$$

$$= 3 \times \sqrt{104} = 3 \times 10{\cdot}2$$

$$= \underline{30{\cdot}6}\text{ in.}$$

$$\text{The minor radius DF} = AF - AD$$

$$\text{and AD} = \sqrt{(2^2 + 10^2)}$$

$$= 10{\cdot}2\text{ in.}$$

$$\therefore DF = 30{\cdot}6 - 10{\cdot}2 = \underline{20{\cdot}4}\text{ in.}$$

The properties of a right angled triangle may also be applied to the calculations of areas and weight of hoods and hoppers with flat sloping sides.

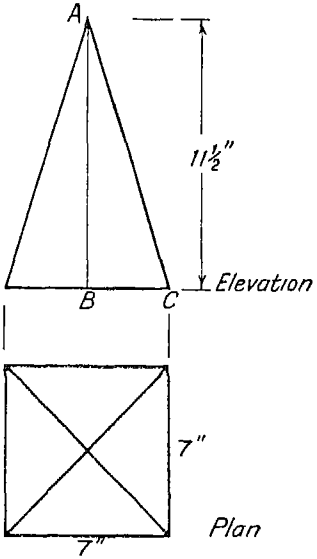


FIG. 23

EXAMPLE 11. *Calculate the weight of metal used in the construction of 100 tapered pyramids like the one in Fig. 23 if 3% of the metal used is allowed for laps and the material weighs 5 lb/sq. ft.*

Fig. 23 shows that all the sides of a pyramid are triangles similar to each other. In this example the vertical height of the pyramid is given and the true height of one of the triangles must first be calculated before the area can be found. Hence, the true height of one of the plates will be equal to the slant side of the pyramid, shown in the elevation, and by drawing a vertical line from the apex to the base a right angled triangle will be formed.

Then the required true length

$$AC = \sqrt{(AB^2 + BC^2)} = \sqrt{(11\tfrac{1}{2}^2 + 3\tfrac{1}{2}^2)}$$
$$= \sqrt{(132{\cdot}25 + 12{\cdot}25)} = \sqrt{144{\cdot}5}$$
$$= 12{\cdot}02 \text{ in.}$$

$$\text{The area of one triangle} = \frac{a \times b}{2} = \frac{7 \times 12{\cdot}02}{2}$$

$$\text{The area of the pyramid} = \frac{7 \times 12{\cdot}02 \times 4}{2} = 168{\cdot}28 \text{ sq. in.}$$

$$\text{Total area of metal used} = \frac{168{\cdot}28}{144} \times 100 \times \frac{103}{100}$$ (Divide by 144 to change sq. in. to sq. ft.)

$$= 120{\cdot}4 \text{ sq. ft.}$$
$$\text{Weight of metal} = 120{\cdot}4 \times 5$$
$$= \underline{602 \text{ lb.}}$$

EXAMPLE 12. *A tapered symmetrical hood, with an open top 15 in. long and 5 in. wide and a bottom which is 30 in. long and 20 in. wide, has a uniform vertical height of 12 in. Calculate (a) the weight of the hood when made from metal weighing 5·2 lb/sq. ft, (b) the length of welding required if the hood is made in four pieces and welded at the corners.*

Fig. 24(a) shows that all the sides are trapeziums, the two long sides are similar and also the two ends. Thus the true depth of the plates will be equal to the slant height AC as shown in the elevation, and this length must be calculated before the true area may be found.

$$\text{Then the required true length } AC = \sqrt{(AB^2 + BC^2)}$$
$$= \sqrt{(7{\cdot}5^2 + 12^2)} = \sqrt{(56{\cdot}25 + 144)}$$
$$= \sqrt{200{\cdot}25} = 14{\cdot}15 \text{ in.}$$

The ends will be made up of two trapeziums as shown, and the area will be equal to:

$$\text{Area} = a \times \frac{b+c}{2} = 14{\cdot}15 \times \frac{20+5}{2}$$

$$= 14{\cdot}15 \times 12{\cdot}5 = 176{\cdot}875 \text{ sq. in.}$$

$$= 353{\cdot}75 \text{ sq. in. for both ends.}$$

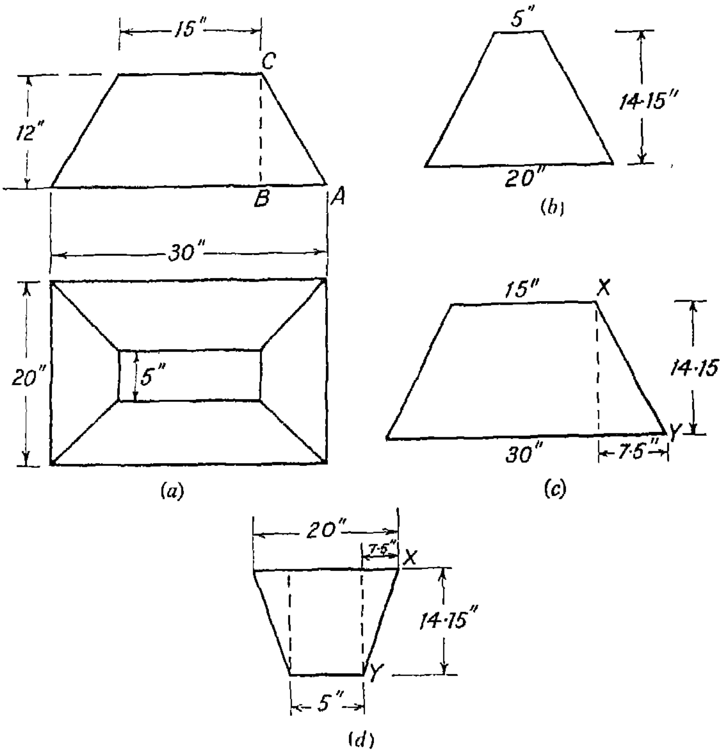


Fig. 24

Similarly the area of the side plates will be equivalent to two trapeziums with the same altitude.

$$\text{Area} = a \times \frac{b+c}{2} = 14{\cdot}15 + \frac{30+15}{2}$$

$$= 14{\cdot}15 \times 22{\cdot}5 = 318{\cdot}375 \text{ sq. in.}$$

$$= 636{\cdot}75 \text{ sq. in. for both sides.}$$

Therefore the weight of the hopper will be:

$$\left(\frac{353{\cdot}75 + 636{\cdot}75}{144}\right) \times 5{\cdot}2$$

$$= 6{\cdot}878 \times 5{\cdot}2 = \underline{35{\cdot}77 \text{ lb.}}$$

The Theorem of Pythagoras is used again to find the length of welding to be carried out, by dividing one of the templates into the right angled triangle shown in Fig. 24(d) and calculating the length XY, which is the true length of metal at the edge of the trapezium.

Then $XY = \sqrt{(14{\cdot}15^2 + 7{\cdot}5^2)} = \sqrt{(200{\cdot}25 + 56{\cdot}25)}$

$$= \sqrt{256{\cdot}5} = 16{\cdot}02 \text{ in.}$$

$$= \underline{64{\cdot}08} \text{ in. for four corners.}$$

EXAMPLE 13. *An open topped corner wall bin has dimensions shown in Fig. 25(a). Calculate the area of metal in the bin, which is closed at the bottom.*

The sides of the hopper are made of four trapeziums and the plan view shows that sides AB, 1 : 2, and AD, 1 : 4, are vertical, whilst sides BC, 2 : 3, and CD, 3 : 4, are inclined. Thus, the true depth of sides AB, 1 : 2, and AD, 1 : 4, is equal to the vertical depth of the bin and their surface areas are obtained as follows:

Plate AB, 1:2 $\quad \text{area} = a \times \left(\frac{b + c}{2}\right)$

$$= 3 \times \frac{5 + 2}{2} = 3 \times 3{\cdot}5$$

$$= 10{\cdot}5 \text{ sq. ft.}$$

Plate AD, 1:4 $\quad \text{area} = 3 \times \frac{3 + 1}{2} = 3 \times 2$

$$= 6 \text{ sq. ft.}$$

The true depth of the plate BC, 2: 3, will be equal to the slant height of the bin as shown in the elevation, Fig. 25(b), and by drawing a line from the bottom corner to the top edge a right angled triangle is formed.

$$\text{The required true depth B, 2} = \sqrt{[3^2 + (5 - 2)^2]} = \sqrt{(3^2 + 3^2)}$$

$$= \sqrt{18} = 4{\cdot}243 \text{ ft.}$$

Plate BC, 2:3

$$\text{Area} = 4{\cdot}243 \times \frac{3 + 1}{2} = 4{\cdot}243 \times 2 = 8{\cdot}486 \text{ sq. ft.}$$

Similarly, the slant height of the plate CD, 3·4, is equal to the slant height D4 shown in the projected elevation, Fig. 25(c), where a right angled triangle has again been formed by drawing a line from the bottom corner to intersect the top edge.

Then, the true depth $D4 = \sqrt{[3^2 + (3 - 1)^2]} = \sqrt{(3^2 + 2^2)} = \sqrt{13} = 3{\cdot}606$ ft.

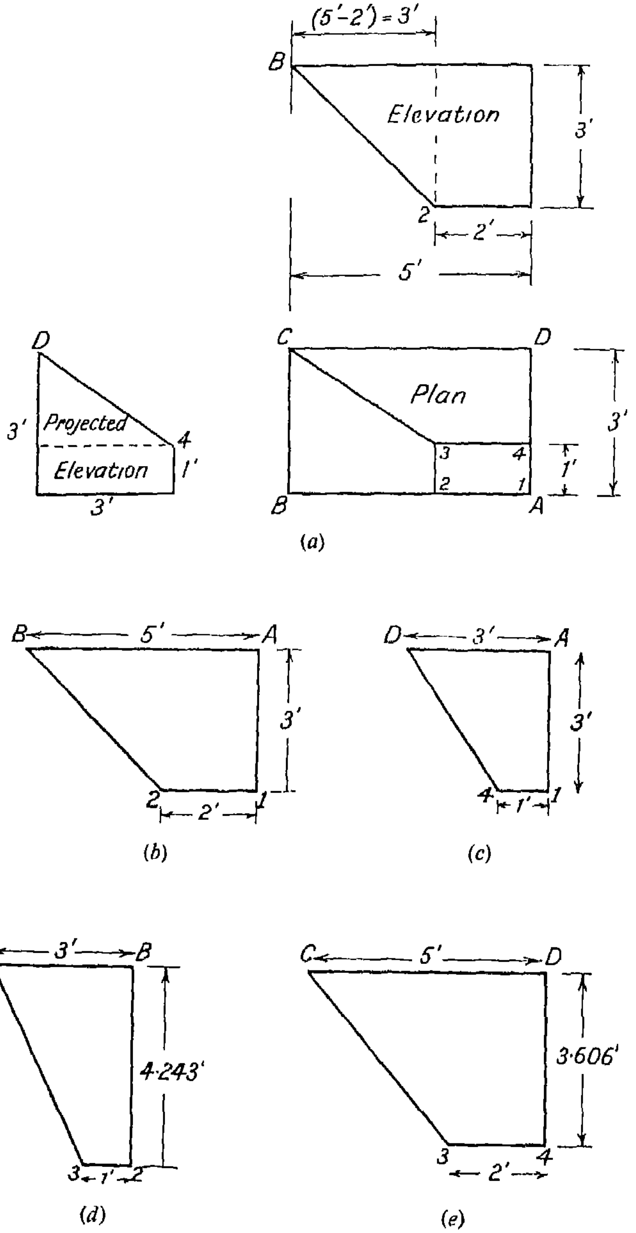
Plate CD, 3:4 $\quad \text{Area} = 3{\cdot}606 \times \frac{5 + 2}{2} = 3{\cdot}606 \times 3{\cdot}5$

$$= 12{\cdot}621 \text{ sq. ft.}$$

$$\text{Area of bottom} = 2 \times 1 = 2 \text{ sq. ft.}$$

$$\text{Total area of metal in bin} = 10{\cdot}5 + 6 + 8{\cdot}486 + 12{\cdot}621 + 2 \text{ sq. ft.}$$

$$= \underline{39{\cdot}607 \text{ sq. ft.}}$$


(5′-2′) = 3′
B
Elevation
3′
2
2′
5′
C
D
Plan
3′
3
4
2
1
1′
B
A
D
3′
Projected
Elevation
4
1′
3′
B
5′
A
3′
2
1
2′
D
3′
A
3′
4
1′
1
C
3′
B
4·243′
3
1′
2
C
5′
D
3·606′
3
4
2′

(a)

(b) (c)

(d) (e)

Fig. 25

Exercise 9

1. A sheet metal tray 3 ft 2 in. long by 1 ft 5 in. wide and $2\frac{1}{2}$ in. deep is to be made with an open top and wired round the edges. If the wiring allowance is to be $\frac{1}{4}$ in., calculate how many sq. ft. of metal will be used in making the tray.
2. An open-topped water tank 4 ft 6 in. long by 2 ft $7\frac{1}{2}$ in. wide and 1 ft 9 in. deep is to be made from metal weighing 5·2 lb/sq. ft. When in use the total weight of the tank and the water it contains is 900 lb. Calculate (*a*) the weight of water in the tank, (*b*) the percentage of the total weight taken up by the metal.
3. A press tool is used for blanking out the shape of sheet metal boxes to be made in one piece. If the finished sizes of the box are to be 5·2 in. long by 3·7 in. wide by 2·4 in. deep, calculate the percentage of scrap made by notching the corners during the blanking operation.
4. A sheet metal apprentice was asked to cut out a quantity of 6 in. squares of metal. Due to incorrect setting of the guillotine the shapes were in the form of a rhombus with an altitude of 5·89 in. Calculate the percentage of error in area of the plates.
5. A fume extraction hood is made of four equal plates each in the form of a trapezium. Calculate the weight of the hood if one plate measures 13 in. at the top, 3 ft 3 in. at the bottom and 2 ft 6 in. vertical depth. Assume the metal weighs 5·4 lb/sq. ft.
6. Determine graphically the minimum diameter of a main air duct to take up the flow from three branch pipes 2·6, 3·75 and 4 in. diameter respectively.
7. Calculate the radius of the pattern for a right cone 12·7 in. base diameter and 23 in. vertical height.
8. A right cone frustum is 3 ft diameter at the bottom, 5 ft 6 in. diameter at the top and has a vertical height of 3 ft. Calculate the major and minor radii of the pattern.
9. A tapered symmetrical hood with an open top and bottom has a uniform vertical height of 10 in. If the bottom is 25 in. long by 18 in. wide, and the top is 20 in. long by 13 in. wide, calculate, (*a*) the weight of the hood if made from metal weighing 5 lb/sq. ft, (*b*) the length of welding required if the hood is made in four pieces and welded at the corners.
10. A straight air pipe enters a building at a position on a wall 15 ft from the ground and 2 ft from the adjacent wall. It leaves the other end of the building, which is 50 ft long, at a position 11 ft from the ground and 9 ft from the adjacent wall. Assuming that the building is rectangular in shape, calculate the length of the pipe.

Circular Measure

Circumference of a Circle

The distance round the perimeter of any circle may be measured by wrapping a strip of paper round it and pricking through the overlapping ends. If the paper is then unrolled and the distance between the pinholes measured, it will be found that the diameter is contained in the perimeter $3\frac{1}{7}$ times.

Hence $$\frac{\text{perimeter}}{\text{diameter}} = \frac{22}{7} = 3\tfrac{1}{7}.$$

This ratio *which is constant for any circle* is called ‘ pi ’. (A Greek letter written π.)

Summarising:

$$\frac{\text{Perimeter}}{\text{Diameter}} = \pi$$

$$\therefore \quad \text{Perimeter} = \pi \times \text{diameter}$$

Now the diameter of a circle is twice the radius, i.e., $d = 2r$.

$$\therefore \quad \text{Perimeter} = 2\pi r$$

where r is the radius of the circle.

The mathematical term for the perimeter of a circle is the **circumference**, and it is more usual to write

$$\text{Circumference} = \pi \times \text{diameter} = \pi d$$

or

$$\text{Circumference} = 2\pi \times \text{radius} = 2\pi r.$$

EXAMPLE 1. *Calculate the length of wire required to wire the tops of* 20 *sheet metal cans if the diameter at the top is* 6 *in.*

$$\text{Circumference} = \pi r = \tfrac{22}{7} \times 6 = 18\tfrac{6}{7} \text{ in.}$$

$$\text{For 20 cans} \quad = 18\tfrac{6}{7} \times 20 = 377\tfrac{1}{7} \text{ in.} = \underline{31 \text{ ft } 5\tfrac{1}{7} \text{ in.}}$$

EXAMPLE 2. *A strip of metal* 44 *in. long is rolled into a circle. Calculate the radius.*

$$\text{Circumference} = 2\pi r$$

$$\therefore \quad r = \frac{\text{circumference}}{2\pi} = \frac{44 \times 7}{2 \times 22}$$

$$\therefore \quad \text{Radius} = \underline{7 \text{ in.}}$$

In practice the dimensions are frequently given on drawings in decimal form, and π may be expressed as $22 \div 7 = 3{\cdot}142$ (to four significant figures or, more usually, $\pi = 3{\cdot}14$).

EXAMPLE 3. *The handle of a spinning jenny is situated at a distance of* 6·32 *in. from the centre of the wheel. How far does it travel if* 15 *revolutions are required to turn up the edge of a blank?*

$$\begin{aligned}
\text{Distance travelled in one rev.} &= 2\pi r = 2 \times 3{\cdot}14 \times 6{\cdot}32 \\
\text{Distance travelled in 15 rev.} &= 2 \times 3{\cdot}14 \times 6{\cdot}32 \times 15 \\
&= 595{\cdot}344 \text{ in.} \\
&= \underline{49 \text{ ft } 7{\cdot}344 \text{ in.}}
\end{aligned}$$

Area of a Circle

The circle of radius r is divided into an equal number of sectors, which are cut out and placed side by side as shown in Fig. 26. It will be seen that the parallelogram formed has a base approximately equal to *half the circumference* and a height approximately equal to the *radius* of the circle.

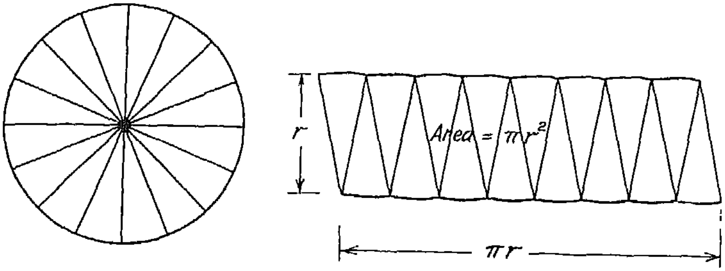


FIG. 26

Then the area of the parallelogram

$$= \text{radius} \times \frac{2\pi r}{2} = \pi r^2.$$

Now the diameter d is twice the radius $\left(r = \frac{d}{2}\right)$

Substituting, area of circle $= \pi\left(\frac{d}{2}\right)^2 = \pi\frac{d^2}{4}$

$$\therefore \quad \text{area of circle} = \pi r^2 \text{ or } \frac{\pi d^2}{4}$$

EXAMPLE 4. *Calculate the area of a circular disc* 8 *in. in diameter.*

(*a*) Area $= \pi r^2 = \pi \times 4^2 = 16\pi = 50{\cdot}24$ sq. in.

(*b*) Area $= \frac{\pi d^2}{4} = \frac{\pi \times 8^2}{4} = \frac{\pi \times 64}{4} = 16\pi = \underline{50{\cdot}24}$ sq. in.

EXAMPLE 5. *A circular end plate weighs* 20½ *lb. Calculate the diameter of the plate if the metal weighs* 2½ *lb/sq. ft* ($\pi = 3{\cdot}14$).

Area of plate $= 20{\cdot}5 \div 2{\cdot}5 = 8{\cdot}2$ sq. ft.

Now, area $= \frac{\pi d^2}{4} \quad \therefore \quad d^2 = \frac{4A}{\pi}$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 8{\cdot}2}{\pi}} = \sqrt{10{\cdot}44}$$

$\therefore$ Diameter $= \underline{3{\cdot}23 \text{ ft.}}$

Area of an Annulus (Fig. 27)

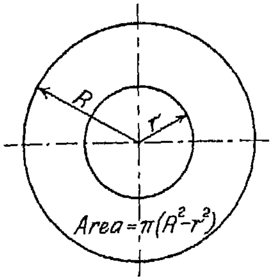


FIG. 27

An annulus is a plane figure contained between two concentric circles, and its area is found by subtracting the area of the smaller circle from the area of the larger circle.

Hence, area = $\pi R^2 - \pi r^2$

This operation may be simplified by taking out the constant π and expressing thus:

$$\text{Area} = \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$$

or alternatively,

$$\text{Area} = \frac{\pi D^2}{4} - \frac{\pi d^2}{4}$$

$$= \frac{\pi}{4}(D^2 - d^2) = 0{\cdot}7854(D^2 - d^2)$$

EXAMPLE 6. *Find the area of an annulus* 12 *in. outside diameter and* 4 *in. inside diameter.*

$$R = 6;\ r = 2$$

(*a*) Area = $\pi(6^2 - 2^2) = \pi(36 - 4) = 32\pi = 100{\cdot}48$ sq. in.

(*b*) alternatively $D = 12;\ d = 4$

Area = $\cdot7854\,(12^2 - 4^2) = \cdot7854\,(144 - 16) = \cdot7854 \times 128 = \underline{100{\cdot}48 \text{ sq. in.}}$ (approx.).

EXAMPLE 7. *From a sheet of mild steel* 6 *ft long and* 3 *ft wide,* 50 *annular discs are to be cut. The outside diameter is* 7 *in. and the internal diameter* 4 *in. Calculate* (*a*) *the weight of the* 50 *tops and* (*b*) *the percentage of scrap from the sheet.* (*The sheet weighs* 0·8 *lb/sq. ft.*)

(*a*) Area of top $= \pi(R^2 - r^2) = \pi(3\tfrac{1}{2}^2 - 2^2)$

$= \pi(12{\cdot}25 - 4) = 8{\cdot}25\pi$ sq. in.

Area of 50 tops $= 8{\cdot}25\pi \times 50$

$= 412{\cdot}5\pi$ sq. in. $= 1296$ sq. in.

Weight of tops $= \dfrac{1296 \times \cdot8}{144} = \underline{7{\cdot}2 \text{ lb.}}$

(*b*) Percentage of scrap $= \dfrac{\text{Area of sheet} - \text{Area used}}{\text{Area of sheet}} \times 100$

Area of sheet $= (6 \times 12) \times (3 \times 12) = 2592$ sq. in.

Percentage scrap $= \dfrac{2592 - 1296}{2592} \times \dfrac{100}{1}$

$= \dfrac{1296}{2592} \times \dfrac{100}{1}$

$= \underline{50\%}.$

Area of a Sector

A sector is any part of a circle bounded by an arc and two radii, Fig. 28. A circle has an included angle of 360°; thus a sector having an angle of 1° will have an area of $\frac{1}{360}$ × area of circle. Similarly if the included angle of any sector is $\theta°$ (pronounced ' theta ') then the area is

$$\frac{\theta}{360} \times \text{area of circle} = \frac{\theta}{360} \times \pi r^2$$

and the length of the arc in any sector

$$= \frac{\theta}{360} \times 2\pi r$$

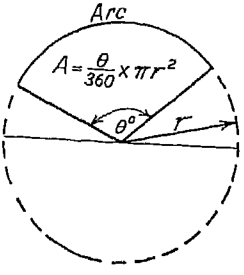


FIG. 28

EXAMPLE 8. *The sector of a circle has radii of* 5 *in. and an included angle of* 150°. *Calculate its area.*

$$A = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{150 \times \pi \times 5^2}{360}$$

$$= \frac{5 \times \pi \times 25}{12} = 32{\cdot}7 \text{ sq. in.}$$

Application to Conic Radii

A common problem occurring in practice is the striking out of the pattern for a right cone too large to set out geometrically. By using a few simple calculations the necessary pattern dimensions can be obtained.

Fig. 29(a) shows the elevation of a right cone of base diameter D and slant height R, which can be calculated by Pythagoras Theorem if the vertical height is given.

If a circle of radius R is drawn, then the circumference will be $2\pi R$ and will represent an angle of 360° on the circle,

i.e., $$2\pi R = 360°$$

then $$\pi = \frac{360°}{2R} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But the measurement required round the circumference of this circle to give a true pattern is πD and will represent an angle of $\theta°$.

i.e., $$\pi D = \theta°$$

then $$\pi = \frac{\theta^\circ}{D} \qquad \cdots\cdots \qquad (2)$$

Equating the two values of π, $$\frac{\theta^\circ}{D} = \frac{360^\circ}{2R}$$

$$\therefore \quad \theta^\circ = \frac{180^\circ D}{R} \qquad \cdots\cdots \qquad (3)$$

Where θ = included angle of sector.
D = diameter of base of the cone.
R = slant height of cone.

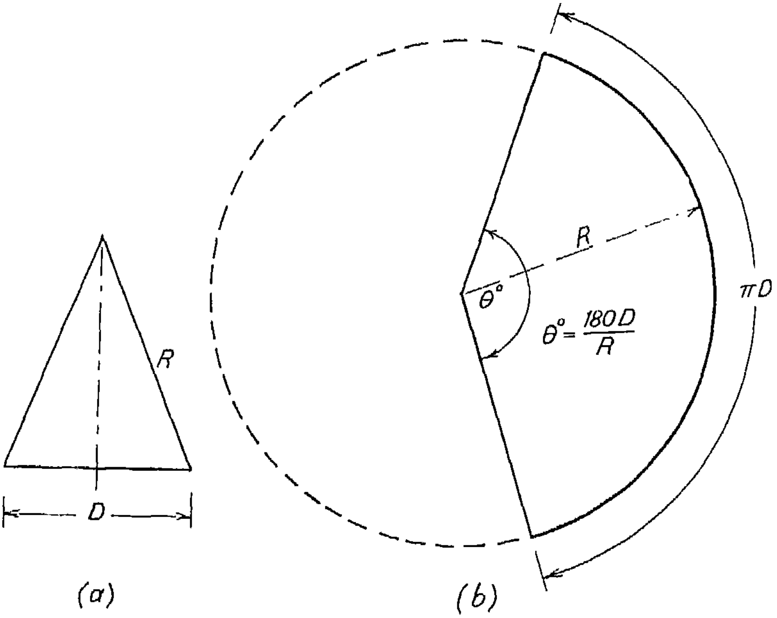


FIG. 29

Summarising:

(*a*) Included angle of pattern $= \theta^\circ = \dfrac{180D}{R}$.
(*b*) Radius of pattern $= R$.
(*c*) Length of pattern arc $= \pi D$.

Where D = base diameter of cone.
R = slant height of cone.

EXAMPLE 9. *A right cone is 9 in. diameter at the base and 6 in. vertical height. Calculate the included angle of the pattern and all other necessary dimensions.*

$$\begin{aligned}\text{Slant height AC} &= \sqrt{(\text{AD}^2 + \text{CD}^2)}\\ &= \sqrt{(4\tfrac{1}{2}^2 + 6^2)} = \sqrt{(20{\cdot}25 + 36)}\\ &= \sqrt{(56{\cdot}25)} = 7\tfrac{1}{2}\text{ in.}\end{aligned}$$

$$\begin{aligned}\text{Included angle of pattern } \theta &= \frac{180 \times D}{R}\\ &= \frac{180 \times 9}{7{\cdot}5} = \underline{216^\circ}\end{aligned}$$

$$\begin{aligned}\text{Circumference of base} &= \pi \times D\\ &= 3{\cdot}14 \times 9 = \underline{28{\cdot}26\text{ in.}}\end{aligned}$$

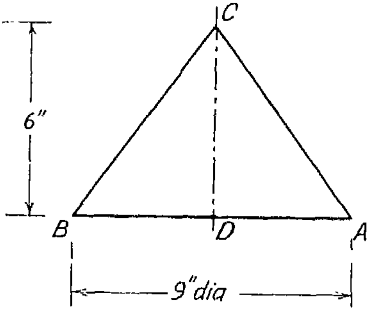


FIG. 30

The Ellipse

The area of an ellipse is found by multiplying the product of the semi-diameters by the constant π.

Thus, Area of ellipse $= \pi Rr$.

Where R is the major radius

and r is the minor radius.

The approximate circumference, or length of the perimeter, is found by multiplying the sum of the semi-diameters by the constant π,

i.e., approximate perimeter of ellipse $= \pi(R + r)$.

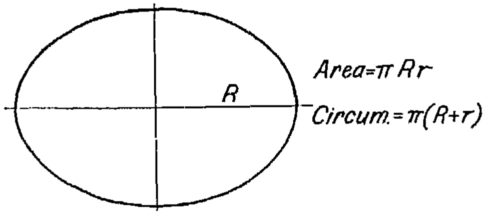


FIG. 31

More accurately, perimeter $= \pi\sqrt{2(R^2 + r^2)}$

EXAMPLE 10. *A pipe 6 in. diameter is cut diagonally (Fig. 32) and an end plate is welded to it. Calculate (a) the end area of the pipe at this section, and (b) the length of welding required to join the plate and pipe together.* ($\pi = 3{\cdot}14$.)

$$\begin{aligned}\text{Area of plate} &= \pi Rr = \pi \times 3 \times 4{\cdot}2\\ &= \underline{39{\cdot}564 \text{ sq. in.}}\\ \text{Length of welding} &= \pi\sqrt{2(R^2 + r^2)}\\ &= \pi\sqrt{2(4{\cdot}2^2 + 3^2)} = \pi\sqrt{53{\cdot}28}\\ &= \underline{22{\cdot}9 \text{ in.}}\end{aligned}$$

EXAMPLE 11. *A petrol container with elliptical ends holds* 4000 *lb of petrol. If the ellipse measures* 12 *in. by* 8 *in., calculate the pressure in lb/sq. in. on the bottom when the tank is upright* (*Fig.* 33).

$$\begin{aligned}\text{Area of ellipse} &= \pi Rr = \pi \times 6 \times 4\\ &= 24\pi = 75{\cdot}36 \text{ sq. in.}\\ \text{Weight or force} &= 4000 \text{ lb.}\\ \text{Pressure} &= \frac{\text{force lb}}{\text{area in.}^2} = \frac{4000}{75{\cdot}36} = \underline{53 \text{ lb/sq. in.}}\end{aligned}$$

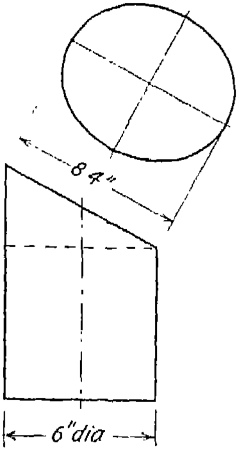


FIG. 32

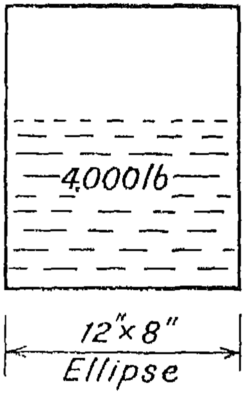


FIG. 33

Applications of Mensuration of Plane Figures to Hoppers and Hoods

Calculation involving areas of plane figures may be applied to ductwork systems, thus enabling them to carry the correct volume of air and maintain a steady rate of flow. The sum of the areas of the outlet pipes should always be equal in area to the main duct, neglecting frictional losses. Hoods and hoppers in the form of tallboy transformers are often required to possess inlets and outlets which are in direct proportion, and here again all calculations are based on areas and not on diameters or lengths of sides.

EXAMPLE 12. *A main air duct of* 9 *in. diameter is to take up the flow from two equal diameter branches. Calculate the diameter of the branches if* 10% *of the available area is to be allowed for frictional losses.*

$$\begin{aligned}
\text{Area of main} &= \pi R^2 = \pi \times 4{\cdot}5^2 \\
&= 20{\cdot}25\pi = 63{\cdot}59 \text{ sq. in.} \\
\text{Frictional allowance} &= 63{\cdot}59 \times \frac{10}{100} \\
&= 6{\cdot}359 \text{ sq. in.} \\
\text{Available area} &= 63{\cdot}59 - 6{\cdot}359 \\
&= 57{\cdot}23 \text{ sq. in.} \\
\text{Area of each branch} &= 57{\cdot}23 \div 2 \\
&= 28{\cdot}61 \text{ sq. in.} \\
\text{From Area } a &= \frac{\pi d^2}{4}
\end{aligned}$$

$$d = \sqrt{\frac{4a}{\pi}} = \sqrt{\frac{4 \times 28{\cdot}61}{3{\cdot}14}}$$

The diameter of branches = $\underline{6{\cdot}03 \text{ in.}}$

EXAMPLE 13. *A circular duct* 16 *in. diameter has two branch pipes, one of which is* 8 *in. diameter. Calculate the diameter of the second pipe so that the two pipes will carry the same volume of air as the main duct.*

$$\begin{aligned}
\text{Area of main duct} &= \pi R^2 = \pi \times 8^2 \\
&= 64 \times \pi = 200{\cdot}96 \text{ sq. in.} \\
\text{Area of 8 in. branch} &= \pi \times 4^2 = 16\pi \\
&= 50{\cdot}24 \text{ sq. in.} \\
\text{Then area of second pipe} &= 200{\cdot}96 - 50{\cdot}24 \\
&= 150{\cdot}72 \text{ sq. in.} \\
\text{From} \qquad a &= \pi r^2 \\
r &= \sqrt{\frac{a}{\pi}} \\
&= \sqrt{\frac{150{\cdot}72}{3{\cdot}14}} = \sqrt{48} \\
&= 6{\cdot}928 \text{ in.} \\
\text{Then diameter of second branch} &= 6{\cdot}928 \times 2 = 13{\cdot}856 \text{ in.} \\
&\qquad \underline{\text{say } 13{\cdot}86 \text{ in.}}
\end{aligned}$$

Alternatively, because all the pipes are circular the answer may be obtained by the application of Pythagoras Theorem.

$$\begin{aligned}
\text{Hence,} \qquad \pi R^2 &= \pi r_1^2 + \pi r_2^3 \\
\text{Dividing through by } \pi, \quad R^2 &= r_1^2 + r_2^2 \\
\therefore \qquad 8^2 &= 4^2 + r^2_2 \\
64 - 16 &= r^2_2 \\
\text{From which} \qquad r_2 &= \sqrt{48} \\
r_2 &= 6{\cdot}928 \\
\text{and } d_2 &= 13{\cdot}856 \text{ in.} \\
&\quad \underline{\text{say } 13{\cdot}86 \text{ in.}}
\end{aligned}$$

EXAMPLE 14. *A hopper has a circular end of 1 ft 8 in. diameter, and transforms into a square which is three times the area of the circle. Calculate the length of the sides of the square end.*

$$\begin{aligned}
\text{Area of circle} &= \pi R^2 \\
&= \pi \times 10^2 = 100\pi \\
&= 314 \text{ sq. in.} \\
\text{Area of square} &= 314 \times 3 = 942 \text{ sq. in.} \\
\text{Length of sides} &= \sqrt{942} = 30{\cdot}69 \text{ in.} \\
&= \underline{2 \text{ ft } 6{\cdot}69 \text{ in.}}
\end{aligned}$$

EXAMPLE 15. *A fume extraction hood has a 7 in. diameter circular end which transforms to a rectangle of four times the area of the circle. The ratio of the length of the sides of the rectangle is 3 : 1. Calculate the dimensions.*

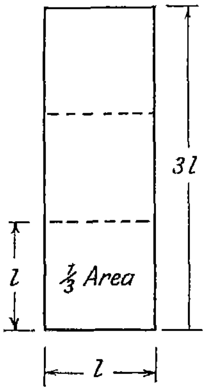


FIG. 34

$$\begin{aligned}
\text{Area of circle} &= \frac{\pi D^2}{4} = \frac{\pi \times 49}{4} \\
&= 38{\cdot}46 \text{ sq. in.} \\
\text{Area of rectangle} &= 38{\cdot}46 \times 4 \\
&= \underline{153{\cdot}84 \text{ sq. in.}}
\end{aligned}$$

Fig. 34 shows a rectangle of sides l and $3l$ and is divided into three squares of equal area. Since the total area of these squares is equal to 153·84 sq. in., then each square has an area of $153{\cdot}84 \div 3$ sq. in.

$$\begin{aligned}
\therefore\ \text{Area of square of side } l &= 51{\cdot}28 \text{ sq. in.} \\
\text{Then}\qquad l &= \sqrt{51.28} \\
\therefore\ \text{Width } l \text{ of rectangle} &= 7{\cdot}16 \text{ in.} \\
\text{Length of rectangle} &= 7{\cdot}16 \times 3 = \underline{21{\cdot}48 \text{ in.}}
\end{aligned}$$

Exercise 10

1. A roll of wire 120 ft long is used to wire the tops of 50 pans of 8½ in. diameter. Calculate the length of wire left over when the tops have been wired.
2. 15 circular blanks of 4·5 in. diameter and 22 of 5·7 in. diameter are cut from a square sheet of copper. Calculate the original dimensions of the sheet, if the remaining metal weighs 15·7 lb and copper weighs 4·7 lb per sq. ft.
3. A circle cutting machine has a cutting wheel of 1·27 in. diameter. How many revolutions will this wheel make in cutting out a circular plate with an area of 2 sq. ft?
4. From a length of steel strip 6 ft long and 3·2 in. wide, 23 washers of 3 in. outside diameter and 2 in. inside diameter are to be punched. Calculate (*a*) the weight of the washers if the metal weighs 5·4 lb/sq. ft and (*b*) the percentage of scrap metal.
5. A sector of a circle has a radius of 12·7 in. and an included angle of 200°. Calculate the radius of a circle having the same area.

6. Calculate all the necessary dimensions and the included angle of the pattern required for a right cone of 9 in. diameter base and 14 in. vertical height.
7. A connecting pipe transforms from an ellipse of 6 in. major axis and 4 in. minor axis into a circle of equal cross-section area. Calculate the diameter of the circular end.
8. A rectangle to round hopper has 9 in. diameter at the base. If the top has 3 times the cross-sectional area as the base and the sides of the rectangle are in a ratio of 2 : 1, calculate the dimensions of the rectangle.
9. A square section main duct pipe is replaced with a circular pipe which carries 30 per cent more air than the original pipe. If the new duct has 10 in. diameter, calculate the lengths of the sides of original pipe.
10. A 24-in. diameter main duct pipe divides into 3 branches, 2 circular and one square. The square branch pipe takes up one half of the available area, whilst the areas of the circular pipes are in the ratio of 7 : 4. Calculate (*a*) the dimensions of the outlet pipes, (*b*) the length of metal required to make up each branch. (Ignore joint allowances.)

Volume and Surface Area of Regular Figures

The space occupied by a body is measured in cubic units and is known as the **volume.** For a solid it is the amount of material contained in the body, whilst for a thin hollow container (and many sheet metal products are in the form of hollow containers) the volume is the determination of holding capacity.

Volume of Square and Rectangular Prisms

By drawing a rectangular prism 3 in. long × 2 in. wide × 2 in. deep (Fig. 35) and dividing each face into squares of 1 in. length and breadth, the prism is found to be made up of twelve cubes of 1 in. sides. The length multiplied by the breadth multiplied by the depth is 3 in. × 2 in. × 2 in. = 12 cubic inches. Therefore, the *volume of the prism = length × breadth × depth* and similarly the *volume of a cube = length × length × length = length*^{3}.

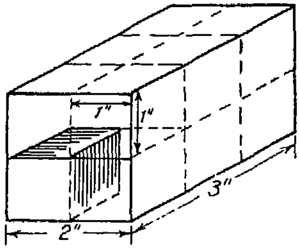


FIG. 35

Again, the term 12 cubic inches has been obtained by multiplying first the units 3 × 2 × 2 = 12 and also the values of the units (inches × inches × inches) = inches^3 or cu. in. Now the length multiplied by the breadth is the end area and therefore

$$\text{volume of a prism} = \text{length} \times \text{breadth} \times \text{depth}$$
$$= \text{end area} \times \text{depth}.$$

The gallon is a unit of capacity.

The following table gives the value of the gallon in cubic inches and other useful information. It should be memorised by the student.

1 cu. yd = 27 cu. ft
1 cu. ft = 1728 cu. in.
1 gallon = 277·4 cu. in.
1 gallon of water = 10 lb
1 cu. in. of water = 0·036 lb
1 cu. ft of water = 62·4 lb

Specific Gravity of a substance is the weight of a given volume of a substance when compared to the weight of an equal volume of water.

EXAMPLE 1. *A rectangular tank* 6 *ft long,* 2 *ft* 6 *in. wide by* 1 *ft* 9 *in. deep is filled with water which is then drained off at the rate of* 2·5 *gallons/minute. How long does it take to empty the tank?*

Volume of tank = end area × depth
= 6 × 2·5 × 1·75 = 26·25 cu. ft
Capacity of tank = 26·25 × 6·24 = 163·8 gallons
Time to empty = 163·8 ÷ 2·5 = 65·52 minutes

EXAMPLE 2. 100 *gallons of water are poured into a quenching tank used for heat treatment. If the tank is* 3 *ft* 2 *in. by* 1 *ft* 7 *in. by* 3 *ft* 6 *in. deep, how far is the water level below the top of the tank?*

Now, 100 gallons = 100 × 277·4 cu. in.
= 27740 cu. in.
Volume = end area × depth

$$\therefore \quad \text{Depth} = \frac{\text{volume}}{\text{end area}} = \frac{27740}{38 \times 19} = 38{\cdot}4 \text{ in.}$$

Distance from top = 42 − 38·4 = 3·6 in.

EXAMPLE 3. *A block of copper weighs* 4 *lb and is to be rolled into a square sheet* 0·028 *in. thick. Calculate the lengths of the sides of the finished sheet. Specific gravity of copper* = 8·95.

1 cu. in. of water = 0·036 lb
∴ 1 cu. in. of copper = 0·036 × 8·95 = 0·322 lb.

$$\text{Cubic capacity of metal} = \frac{4}{{\cdot}322} = 12{\cdot}422 \text{ cu. in.}$$

Volume of metal = area × thickness

$$\therefore \quad \text{Area} = \frac{\text{volume}}{\text{thickness}} = \frac{12{\cdot}422}{0{\cdot}028}$$

= 443·6 sq. in.

Length of side = $\sqrt{443{\cdot}6}$ = 21·06 in. = 1 ft 9·06 in.

Note that from the formula, Area $= \dfrac{\text{Volume}}{\text{Thickness}}$ the answer 443·6 sq. in. was obtained by first dividing the units $(12{\cdot}422 \div 0{\cdot}028) = 443{\cdot}6$ and then dividing the values of the units

$$\frac{\text{cubic in.}}{\text{in.}} = \frac{\text{in.} \times \text{in.} \times \text{in.}}{\text{in.}} = \text{in.}^2$$

Hence area = $\underline{443{\cdot}6 \text{ sq. in.}}$

EXAMPLE 4. *An open rectangular tank is* 4 *ft long,* 3 *ft wide and* 18 *in. deep. It is lined with sheet lead* ⅛ *in. thick.*

(*a*) *Calculate the weight of lead used, allowing* 12½% *for flanges, etc. and*
(*b*) *assuming the tank is filled to within* 4 *in. of the top with water containing* 10% *hydrochloric acid, calculate the weight of the solution.*

(1 *cu. in. of lead* = 0·41 *lb;* 1 *cu. in. of water* = 0·036 *lb; specific gravity of* 10% *hydrochloric solution* = 1·05.)

Perimeter = 4 + 4 + 3 + 3 = 14 ft
Area of sides = 14 × 1·5 = 21 sq. ft
Area of base = 4 × 3 = 12 sq. ft
Area of metal in tank = 21 + 12 = 33 sq. ft
Volume of metal in tank $= 33 \times \overset{18}{\cancel{144}} \times \dfrac{1}{\cancel{8}} = 594$ cu. in.
(sq. in. = sq. ft × 144).
Allowances for flanges etc. $= 594 \times \dfrac{12{\cdot}5}{100} = 74{\cdot}25$ cu. in.
Volume of metal used = 594 + 74·25 = 668·25 cu. in.
Weight of lead used = 668·25 × 0·41 = $\underline{273{\cdot}98 \text{ lb}}$
Volume of tank = end area × depth
Now, depth of liquid = 18 − 4 = 14 in. = $1\frac{1}{6}$ ft
Volume $= 4 \times 3 \times 1\frac{1}{6} = \overset{2}{\cancel{4}} \times \overset{1}{\cancel{3}} \times \dfrac{7}{\underset{2}{\cancel{6}}} = 14$ cu. ft
Weight of solution = volume × weight/cu. ft × sp. gr.
= 14 × 1728 × ·036 × 1·05
(1 cu. ft = 12 × 12 × 12 = 1728 cu. in.).
= $\underline{914{\cdot}45 \text{ lb.}}$

Volume of a Cylinder

The formula *end area multiplied by the depth* holds good for the volume of any right prism. Hence the volume of a cylinder which has a circular cross-section is found as follows:

$$\begin{aligned}\text{Volume of cylinder} &= \text{end area} \times \text{depth} \\ &= \pi r^2 \times l\end{aligned}$$

where r is the radius and l is the length or depth of the prism.

$$\text{Alternatively, volume} = \text{end area} \times \text{depth}$$
$$= \frac{\pi d^2}{4} \times l$$

where d is the diameter of the cylinder.

EXAMPLE 5. *A cylindrical pan 8 in. diameter at the base and 4 in. deep is filled with water. How many pints of water are contained in the pan?*

Volume of pan $= \pi r^2 l = \pi \times 4^2 \times 4 = 64\pi = 200{\cdot}96$ cu. in.

1 gallon of water $= 277{\cdot}4$ cu. in.

1 pint of water $= 277{\cdot}4 \div 8 = 34{\cdot}67$ cu. in.

Pan holds $= \dfrac{200{\cdot}96}{34{\cdot}67} = 5{\cdot}793$ pints.

say 5·79 pints.

Curved Surface of a Cylinder

The area of metal contained in the curved surface of a cylinder is obtained by multiplying the circumference by the length of the side (Fig. 36).

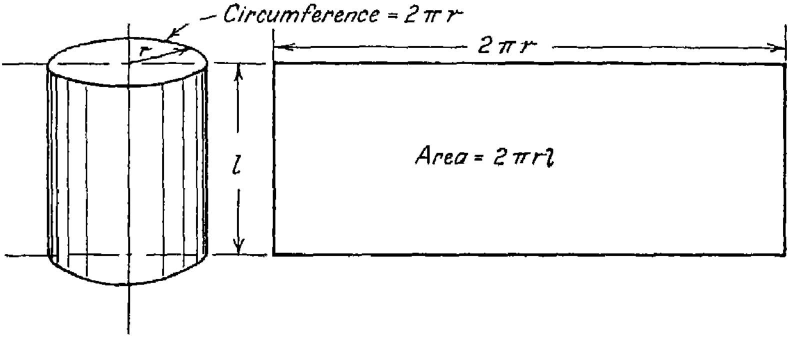


FIG. 36

$$\text{Circumference} = 2\pi r$$
$$\therefore \text{Curved surface area} = 2\pi r l \qquad (1)$$

where r = radius of the cylinder

l = length of the side.

Many sheet metal products are in the form of open-topped containers and for these and similar bodies the base area must be taken into consideration for the purpose of calculation. Hence,

surface area of open-topped cylindrical container

$$= \text{curved surface area} + \text{area of base}$$
$$= 2\pi r l + \pi r^2$$
$$= \pi r(2l + r) \qquad (2)$$

Similarly, when the total area of a closed hollow cylinder is to be calculated, then the areas of both ends must be considered,

i.e., surface area of closed hollow cylinder

$$= \text{curved surface area} + \text{area of both ends}$$
$$= 2\pi rl + 2\pi r^2$$
$$= 2\pi r(1 + r).$$

These formulae may be expressed in terms of the diameter (d) as shown.

$$r = \frac{d}{2}$$

$$\therefore \text{curved surface area} = 2\pi rl = 2\pi \frac{d}{2} l = \pi dl \quad . \quad . \quad . \quad . \qquad (3)$$

Surface area of open-topped cylindrical container $= 2\pi rl + \pi r^2$

$$= 2\pi \frac{d}{2} l + \pi\left(\frac{d}{2}\right)^2$$
$$= \pi dl + \frac{\pi d^2}{4}$$
$$= \pi d\left(l + \frac{d}{4}\right) \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

Surface area of closed hollow cylinder

$$= 2\pi rl + 2\pi r^2$$
$$= 2\pi \frac{d}{2} l + 2\pi\left(\frac{d}{2}\right)^2$$
$$= \pi dl + \frac{\pi d^2}{2} = \pi d\left(l + \frac{d}{2}\right) \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

EXAMPLE 6. *A 'cheese' float in the form of a closed hollow cylinder is required to weigh* 10 *lb. Calculate the weight of ballast to be added to the float before the ends are sealed, if the float's dimensions are* 10 *in. dia. and* 4 *in. deep, and the metal weighs* 5 *lb/sq. ft.*

Surface area of cylinder $= \pi d\left(l + \frac{d}{2}\right) = 10\pi(4 + 5)$

$= 90\pi = 282{\cdot}6$ sq. in.

Weight of float $= \frac{282{\cdot}6 \times 5}{144} = 9{\cdot}82$ lb

Additional weight $= 10 - 9{\cdot}82 = \underline{0{\cdot}18 \text{ lb.}}$

EXAMPLE 7. *An open-top tank is 12 in. by 12 in. by 18 in. deep.*

(*a*) *Calculate the capacity of the tank in gallons.*
(*b*) *Calculate the diameter of a cylindrical tank of equal capacity and having the same depth.*
(*c*) *Compare the weights of the tanks if each is made of sheet metal* 0·024 *in. thick.*

(1 *cu. in. of metal* = 0·3 *lb;* 1 *cu. ft* = 6·24 *gallons.*)

(*a*) Volume of tank = length^2 × depth

= $1^2 \times 1{\cdot}5 = 1{\cdot}5$ cu. ft

Capacity = $1{\cdot}5 \times 6{\cdot}24 =$ 9·36 gallons.

(*b*) Volume of tanks = $1{\cdot}5 \times 1728 = 2592$ cu. in.

$$\text{Volume of cylinder} = \frac{\pi d^2}{4} \times l$$

$$\therefore \quad d = \sqrt{\frac{4v}{\pi l}} = \sqrt{\frac{4 \times 2592}{\pi \times 18}} = \underline{13{\cdot}54 \text{ in.}}$$

(*c*) Square tank

Perimeter = $4 \times 12 = 48$ in.

Area of metal in sides = $48 \times 18 = 864$ sq. in.

Area of metal in base = $12 \times 12 = 144$ sq. in.

Total area = 1008 sq. in.

Volume of metal = area × thickness

Weight of metal = area × thickness × wt/cu. in.

= $1008 \times 0{\cdot}024 \times 0{\cdot}3$

= 7·2576 lb

say 7·26 lb.

Cylindrical tank

$$\text{Area of metal in open-top cylinder} = \pi d\left(l + \frac{d}{4}\right)$$

$$= 13{\cdot}54\pi\left(18 + \frac{13{\cdot}54}{4}\right)$$

$$= 13{\cdot}54\pi(21{\cdot}385) = 909{\cdot}3 \text{ sq. in.}$$

Weight of metal = $909{\cdot}3 \times 0{\cdot}024 \times 0{\cdot}3$

= 6·547 lb

say 6·55 lb.

Miscellaneous Prisms

Fig. 37 shows an elliptical-ended petrol tank of length *l*. Then the volume will be

$$\text{end area} \times \text{length} = \pi Rr \times l = \pi Rrl$$

The curved surface area is found by multiplying the perimeter by the length

$\therefore$ Curved surface area $= \pi(R + r) \times l = \pi l(R + r)$.

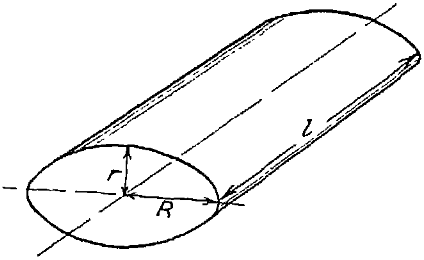


Fig. 37

If the tank has an open end then the surface area

$$= \text{area of curved surface} + \text{area of one end}$$
$$= \pi l(R + r) + \pi(Rr)$$

and for a closed vessel the surface area

$$= \text{area of curved surface} + \text{area of two ends}$$
$$= \pi l(R + r) + 2\pi(Rr).$$

By similar reasoning the formula for the volume and surface area of a hexagonal prism may be derived as

volume = end area × length

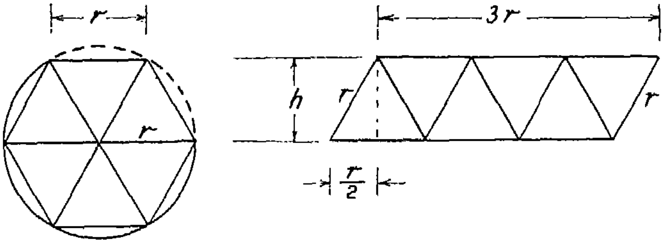


Fig. 38

Now the lengths of the sides of the six equilateral triangles, which make up the end area, are equal to the radius of the circumscribing circle. If these triangles are placed side by side to form the parallelogram shown in Fig. 38, then the length of the base will be three times the length of one of the flat sides of the hexagon, and the altitude of the parallelogram will be equal to the vertical height of one triangle. Thus by the Theorem of Pythagoras

$$r^2 = \left(\frac{r}{2}\right)^2 + h^2$$

$$h^2 = r^2 - \left(\frac{r}{2}\right)^2$$

$$h = \sqrt{\left(r^2 - \frac{r^2}{4}\right)} = \sqrt{\frac{3}{4}r^2} = \frac{\sqrt{3}}{2}r$$

Area of parallelogram $= 3r \times \frac{\sqrt{3}r}{2} = 2{\cdot}598\ r^2$.

Then the volume = end area × length $= 2{\cdot}598\ r^2l$,

where r = length of one of the flat sides of the hexagon.

The surface area of the walls of a hexagonal prism is again the perimeter multiplied by the length,

$$\therefore \text{ surface area} = 6\ rl$$

and if the ends are taken into consideration the necessary additions must be made

i.e., For vessel with open top,

$$\text{surface area} = 6\ rl + 2{\cdot}598\ r^2.$$

For closed vessel,

$$\begin{aligned}\text{surface area} &= 6\ rl + 2 \times 2{\cdot}598\ r^2\\ &= 6\ rl + 5{\cdot}196\ r^2.\end{aligned}$$

Annular Prisms (Fig. 39)

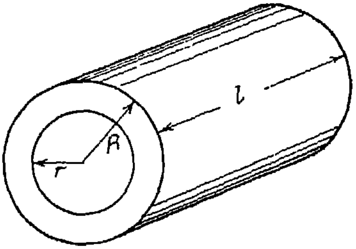


Fig. 39

Floats are often made in the form of closed cylinders with annular ends and the internal volume is calculated from the formula

$$\begin{aligned}V &= \text{end area} \times \text{length}\\ &= \pi(R^2 - r^2)l.\end{aligned}$$

The surface area of metal required to produce such a vessel will be the sum of the areas of the two cylinders plus the two ends,

$$\begin{aligned}\text{i.e., total surface area} &= 2\pi Rl + 2\pi rl + 2\pi(R^2 - r^2)\\ &= 2\pi(Rl + rl + (R^2 - r^2)).\end{aligned}$$

Example 8. *A petrol tank 2 ft 6 in. long has elliptical ends of major axis 12 in. and minor axis 8 in. Calculate (a) the cubic capacity of the tank in gallons, (b) the weight of the tank, if made from metal weighing 4·7 lb/sq. ft.*

$$\begin{aligned}
\text{Volume of tank} &= \text{end area} \times \text{depth} = \pi Rrl \\
&= \pi \times 6 \times 4 \times 30 = 2260{\cdot}8 \text{ cu. in.} \\
\text{Capacity} &= \frac{2260{\cdot}8}{277{\cdot}4} = 8{\cdot}149 \text{ gallons.} \\
&\qquad \underline{\text{say } 8{\cdot}15 \text{ gall.}}
\end{aligned}$$

Area of metal in tank including ends

$$\begin{aligned}
&= \pi l(R + r) + 2\pi Rr = 30\pi(6 + 4) + 2 \times \pi \times 24 \\
&= 300\pi + 48\pi = \pi(300 + 48) \\
&= 348\pi = 1092{\cdot}7 \text{ sq. in.} \\
\text{Weight of tank} &= \text{area in sq. ft} \times \text{wt/sq. ft} \\
&= \frac{1092{\cdot}7}{144} \times 4{\cdot}7 = \underline{35{\cdot}66 \text{ lb.}}
\end{aligned}$$

Example 9. 360 *metal pressings are to be punched from a strip of copper* 0·026 *in. thick. If the pressings are in the form of hexagons with* 2 *in. sides, calculate their weight if the copper weighs* 0·32 *lb/cu. in.*

$$\begin{aligned}
\text{Volume of hexagon} &= \text{end area} \times \text{thickness} \\
&= 2{\cdot}598r^2l \\
&= 2{\cdot}598 \times 4 \times 0{\cdot}026 = 0{\cdot}27 \text{ cu. in.} \\
\text{Weight} &= 360 \times \text{volume} \times \text{wt/cu. in.} \\
&= 360 \times 0{\cdot}27 \times 0{\cdot}32 \\
&= \underline{31{\cdot}11 \text{ lb.}}
\end{aligned}$$

Example 10. 20 *annular floats* 6 *in. outside dia. and* 2 *in. inside dia. by* 10 *in. deep are to be made from copper sheet. Calculate the internal volume of each float and also the percentage of scrap if* 50 *sq. ft of metal is used in the production of the floats.*

$$\begin{aligned}
\text{Volume} &= \text{end area} \times \text{depth} = \pi(R^2 - r^2) \times \text{depth} \\
&= 10\pi(3^2 - 1^2) = 80\pi = \underline{251{\cdot}2 \text{ cu. in.}} \\
\text{Surface area} &= 2\pi\{Rl + rl + (R^2 - r^2)\} \\
&= 2\pi\{(3 \times 10) + (1 \times 10) + (9 - 1)\} \\
&= 2\pi\{30 + 10 + 8\} \\
&= 96\pi = 301{\cdot}44 \text{ sq. in.} \\
\text{Total area} &= \frac{301{\cdot}44 \times 20}{144} = 41{\cdot}86 \text{ sq. ft.} \\
\%\ \text{Scrap} &= \frac{50 - 41{\cdot}86}{50} \times 100 \\
&= \frac{8{\cdot}14 \times \overset{2}{\cancel{100}}}{\cancel{50}} = \underline{16{\cdot}28 \text{ per cent.}}
\end{aligned}$$

Exercise 11

1. A tank 5 ft 3 in. long by 2 ft 2 in. wide holds oil to a depth of 1 ft 4 in. The oil is used to fill a quantity of 1 pint measures. How many containers can be completely filled before the tank is empty?
2. Four storage bins of 2 ft 6 in. diameter and 3 ft 9 in. deep are to be replaced by one larger bin to hold the same contents. If the maximum diameter of the larger bin is to be 4 ft 10 in., calculate (*a*) the depth of this bin, (*b*) the area of metal saved by the replacement. (Assume all the bins to be closed hollow cylinders.)
3. A plate 2 ft long by 1 ft wide by $\frac{1}{2}$ in. thick has a hole 3 in. diameter punched through it, and 15 holes $\frac{7}{8}$ in. diameter drilled at a distance of 1 in. from the sides of the plate. Calculate (*a*) the weight of metal removed, (*b*) the percentage of the sheet lost during the operations of drilling and punching. (1 cu. in. of metal = 0·3 lb.)
4. A tea urn in the form of a cylinder with an open top is made from metal 0·062 in. thick. If the urn is to hold 10 gallons of water and its depth is to be 18 in., calculate (*a*) the diameter of the boiler, (*b*) the weight of the boiler when completely filled with water. (1 cu. in. metal = 0·3 lb.)
5. A metal sheet 3 ft 4 in. long by 2 ft 2 in. wide is rolled into a cylinder 3 ft 4 in. deep. Another sheet of similar dimensions is rolled into a cylinder 2 ft 2 in. deep. Calculate the internal volumes of the cylinders formed.
6. A petrol tank with elliptical ends of 16 in. major axis by 12 in. minor axis by 18 in. long is fitted to a motor car which has a fuel consumption of 38 miles/gallon. If the tank is 75% full at the start of the journey, how far does the car travel before the tank is empty?
7. A bar of steel has a cross-section in the form of a hexagon of 1·5 in. sides. If the bar is 14 in. long, calculate how many yards of wire 0·128 in. diameter can be drawn from it.
8. An annular float 6 in. outside diameter and 3 in. inside diameter is 9 in. deep. Calculate (*a*) the effective volume of the float, (*b*) the weight of the float if made from metal weighing $2\frac{1}{2}$ lb/sq. ft.
9. A cylinder has a diameter of $3\frac{1}{2}$ in. and is partly filled with water. A piece of copper 3 in. × $2\frac{1}{3}$ in. × $10\frac{1}{2}$ in. is completely immersed in the water. How much higher in the cylinder will the water level be raised?
10. An elliptical fuel tank 2 ft 6 in. long has its major and minor axis 1 ft 8 in. and 1 ft respectively. Calculate the weight of petrol in the tank when filled to 80% of its capacity. (1 gallon of petrol = 7·2 lb.)

Right Cones

Experiments have shown that, if a right cone is placed inside a cylinder which has the same dimensions, the cone will occupy one third of the internal volume of that cylinder (Fig. 40).

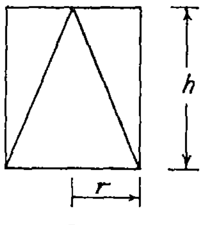


FIG. 40

Let r be the radius of the bases of both the cylinder and cone, and h be their vertical heights,

$$\text{then, volume of cylinder} = \pi r^2 h$$

$$\therefore \quad \text{volume of cone} = \frac{\pi r^2 h}{3}$$

$$\text{or,}\quad \text{volume of cylinder} = \frac{\pi d^2 h}{4}$$

$$\therefore\quad \text{volume of cone} = \frac{\pi d^2 h}{4} \div 3 = \frac{\pi d^2 h}{12}$$

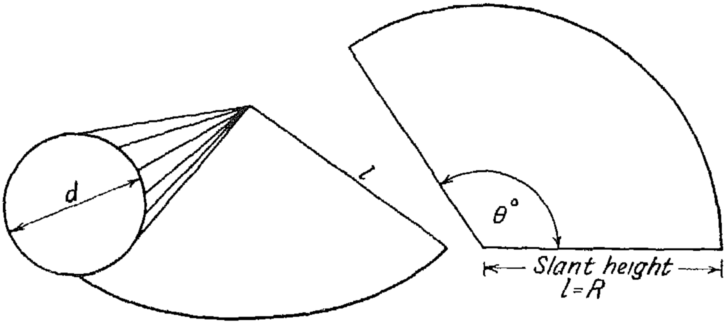


FIG. 41

The surface development of the right cone, Fig. 41, shows that the area of metal in the curved surface of a cone is equal to the area of a sector of a circle with a radius l,

$$\text{i.e.,}\quad \text{curved surface area} = \frac{\theta}{360} \times \pi l^2$$

Now, the included angle $\theta = \dfrac{180d}{R}$ see p. 56, equation (3)

where d = base diameter of cone,

R = slant height.

Substituting for θ, curved surface area $= \dfrac{180d}{360R} \times \pi l^2$

But $l = R$ = slant height,

$$\therefore\ \frac{180d}{360l} \times \pi l^2 = \frac{d}{2} \times \pi l$$

Now, $\dfrac{d}{2} = r$, $\therefore\ d = 2r$,

$\therefore$ curved surface area of right cone $= \pi rl$.

total surface area = curved surface + area of base

$$= \pi rl + \pi r^2$$

$$= \pi r(l + r).$$

Pyramids

The formula $\frac{1}{3}$ (area of base × vertical height) holds good for the volume of any right pyramid.

Hence, volume of square based pyramid $= \dfrac{s^2h}{3}$

and volume of hexagonal based pyramid $= \dfrac{2{\cdot}598s^2h}{3}$

where s is equal to the length of the sides of the base and h is the vertical height.

Similarly, the surface areas of such pyramids are found by multiplying half the perimeter of the base by the slant height of one of the lateral sides,

i.e., surface area of square pyramid $= 2s \times l$

surface area of hexagonal pyramid $= 3s \times l$

where s = length of one side of base,

and l = slant height of pyramid.

EXAMPLE 1. *Calculate the volume and curved surface area of a sheet metal hood in the form of a right cone* 8 *in. diameter at the base and* 10 *in. vertical height.*

$$\text{Volume} = \tfrac{1}{3}\pi r^2h = \tfrac{1}{3}\pi \times 4^2 \times 10$$

$$= \tfrac{1}{3} \times 16 \times 10 \times \pi = 167{\cdot}46 \text{ cu. in.}$$

$$\text{Curved surface area} = \pi rl$$

$$\text{now,} \quad l = \sqrt{(4^2 + 10^2)} = \sqrt{116} = 10{\cdot}77$$

$$\therefore \text{ Curved surface area} = \pi \times 4 \times 10{\cdot}77 = \underline{135{\cdot}27 \text{ cu. in.}}$$

EXAMPLE 2. *Two sheet metal workers are asked to make a tapered container to hold* 100 *cu. in. One decides to produce his container in the form of a cone with a* 6*-in. diameter base, whilst the other decides to produce his container in the form of a hexagonal pyramid with* 3*-in. sides. Calculate the required depth of each container.*

$$\text{Volume of cone} = \tfrac{1}{3}\pi r^2h$$

$$\therefore \ 100 = \tfrac{1}{3}\pi r^2h$$

$$\text{Then } h = \frac{3 \times 100}{\pi r^2} = \frac{\overset{1}{\cancel{3}} \times 100}{\pi \times \underset{3}{\cancel{9}}} = \underline{10{\cdot}61 \text{ in.}}$$

$$\text{Volume of pyramid} = \frac{2{\cdot}598s^2h}{3}$$

$$\therefore \ 100 = \frac{2{\cdot}598s^2h}{3}$$

$$\text{Then } h = \frac{\overset{1}{\cancel{3}} \times 100}{2{\cdot}598 \times \underset{3}{\cancel{9}}} = \underline{12{\cdot}83 \text{ in.}}$$

EXAMPLE 3. *A cylindrical tank of 15 in. diameter base and 14 in. height is to be replaced with one in the form of a cone 20 in. height. Calculate the diameter of the cone if both tanks must have the same volume.*

Now, volume of cylinder = volume of cone

$$\therefore \quad \pi R_1^2 h^1 = \frac{\pi}{3} R_2^2 h_2$$

$$\therefore \quad \pi \times 7\tfrac{1}{2}^2 \times 14 = \frac{\pi \times R_2^2 \times 20}{3}$$

$$\text{Then, } R_2 = \sqrt{\left(\frac{3 \times \pi \times 7\tfrac{1}{2}^2 \times 14}{20 \times \pi}\right)}$$

$$R_2 = \sqrt{118{\cdot}125} = 10{\cdot}86 \text{ in.}$$

$$\therefore \quad \text{Diameter of cone} = \underline{21{\cdot}72 \text{ in.}}$$

Spheres

If a sphere were placed in a cylinder in such a way that the surface of the sphere just touched the base, top and sides of the cylinder, then the sphere would occupy two thirds of the internal volume.

In Fig. 42, let r be the radius of both cylinder and sphere.

$$\text{Then, volume of cylinder} = \pi r^2 h, \text{ and h} = 2r.$$

$$= \pi r^2 \times 2r = 2\pi r^3$$

$$\therefore \text{ volume of sphere} = 2\pi r^3 \times \frac{2}{3} = \frac{4}{3}\pi r^3$$

$$\text{or} \quad \text{volume of cylinder} = \frac{\pi d^2 h}{4}$$

$$\text{volume of sphere} = \frac{\pi d^2 h}{4} \times \frac{2}{3}$$

Now, $h = d$,

$$\therefore \text{ volume of sphere} = \frac{\pi d^2}{4} \times d \times \frac{2}{3} = \frac{\pi d^3}{6}$$

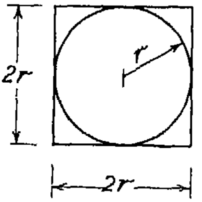


FIG. 42

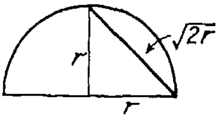


FIG. 43

The radius of blank required to produce a hemisphere is discussed on p. 88 and from here this radius may be calculated. Thus in Fig. 43 the radius of the disc is shown to be $\sqrt{2r^2}$, where r is the radius of the hemisphere,

i.e., area of blank $= \pi \times \sqrt{(2r^2)}^2 = 2\pi r^2$

$\therefore$ area of surface of hemisphere $= 2\pi r^2$

and surface area of surface of sphere $= 2\pi r^2 \times 2 = 4\pi r^2$

Now, $r = \dfrac{d}{2}$

$\therefore$ area of sphere $4\pi\dfrac{d^2}{4} = \pi d^2$.

EXAMPLE 4. *A spherical float 9 in. diameter is made of copper 0·028 in. thick. Calculate the volume and weight of the float. 1 cu. in. of copper weighs 0·32 lb.*

Volume $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 4\frac{1}{2}^3 = \underline{381{\cdot}51 \text{ cu. in.}}$

Surface area $= \pi d^2 = \pi \times 9^2$

$= 81\pi = 254{\cdot}34$ sq. in.

Volume of metal in float $= 254{\cdot}34 \times 0{\cdot}028 = 7{\cdot}12$ cu. in.

Weight of float $= 7{\cdot}12 \times 0{\cdot}32 = \underline{2{\cdot}278 \text{ lb.}}$

EXAMPLE 5. *A boiler consists of a hemispherical base with a vertical cylindrical upper portion. If the diameter of the boiler is 18 in. and its overall depth is 2 ft, calculate the capacity in gallons (Fig. 44).*

Volume of hemisphere $= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times (\frac{3}{4})^3 = 0{\cdot}883$ cu. ft

Volume of cylinder $= \pi r^2 h$

Now, $h = 24 - 9$ $= 15$ in. $= 1\frac{1}{4}$ ft

Volume $= \pi \times (\frac{3}{4})^2 \times 1\frac{1}{4}$

$= \pi \times \frac{9}{16} \times \frac{5}{4} = 2{\cdot}209$ cu. ft

Total volume $= 0{\cdot}883 + 2{\cdot}209 = 3{\cdot}092$ cu. ft

Capacity $= 3{\cdot}092 \times 6{\cdot}24 = \underline{19{\cdot}29 \text{ gallons.}}$

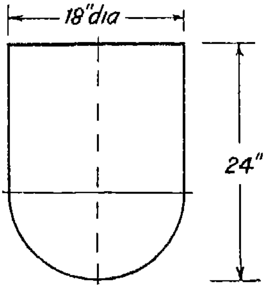


FIG. 44

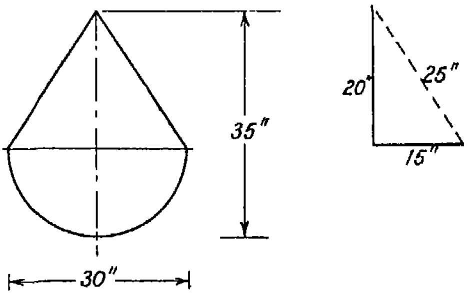


FIG. 45

EXAMPLE 6. *A marine buoy is in the form of a right cone combined to a hemisphere at the base. If the maximum diameter of the buoy is 2 ft 6 in. and the overall height is 2 ft 11 in., calculate the weight of the buoy when made from metal weighing 10 lb/sq. ft (Fig. 45).*

Surface area of hemisphere $= 2\pi r^2 = 2\pi \times 15^2 = 1413$ sq. in.

Slant height of cone $= \sqrt{(15^2 + 20^2)} = \sqrt{(225 + 400)}$

$= \sqrt{625} = 25$ in.

Surface area of cone $= \pi r l = \pi \times 15 \times 25$

$= 1177{\cdot}5$ sq. in.

Total area $= 1413 + 1177{\cdot}5 = 2590{\cdot}5$ sq. in.

Weight of buoy $= \dfrac{2590{\cdot}5 \times 10}{144} = \underline{179{\cdot}9 \text{ lb.}}$

Frustum of Pyramids and Cones

By methods too difficult to prove at this stage it can be shown that the following formulae hold good for the volumes and surface areas of frustums of pyramids, tapered elliptical containers and cones.

(*a*) **Frustum of a pyramid:**

$$\text{Volume} = \frac{1}{3}\, h[A + a + \sqrt{(Aa)}]$$

where a = area of upper surface,

A = area of lower surface,

h = vertical height of frustum.

Surface area = mid-perimeter × slant height

$$= \frac{\text{perimeter of top} + \text{perimeter of base}}{2} \times \text{slant height}$$

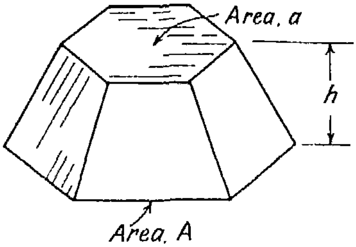


FIG. 46

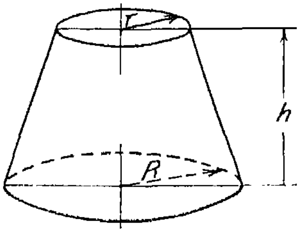


FIG. 47

(*b*) **Frustum of right cone:**

$$\text{Volume} = \frac{1}{3}\,\pi h(R^2 + r^2 + Rr)$$

where r = radius of small end,

R = radius of large end,

h = vertical height of pyramid.

Surface area = mid-circumference × slant height

$$= 2\pi\left(\frac{R+r}{2}\right) \times \text{slant height.}$$

Elliptical Vessels

Volume = (area of large end + area of small end +

$$4 \text{ mid-area}) \times \frac{\text{vertical height}}{6}$$

$$= [A + a + (4 \times \text{mid area})]\,\frac{h}{6}$$

or to the area of both ends add four times the area of the mid-section, multiply by the vertical height and divide by six.

This is known as the **Prismoidal Rule** and gives correct results for any transformers in which both top and bottom are parallel, including frustums of pyramids and cones.

Surface area = mid-perimeter × slant height

$$= \frac{\pi}{2}\,[(R_1 + R_2) + (r_1 + r_2)] \times \text{slant height}$$

where $R_1 = \frac{1}{2}$ top major axis,
$r_1 = \frac{1}{2}$ top minor axis,
$R_2 = \frac{1}{2}$ base major axis,
$r_2 = \frac{1}{2}$ base minor axis.

EXAMPLE 7. *Find the volume and surface area of the frustum of a cone* 18 *in. dia. at the top,* 12 *in. dia. at the base and* 9 *in. vertical height.*

Volume $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$
$= \frac{1}{3} \times \pi \times 9[9^2 + 6^2 + (6 \times 9)]$
$= 3\pi(81 + 36 + 54) = (3\pi \times 171)$
$= \underline{1610{\cdot}8 \text{ cu. in.}}$

Surface area = mid-circumference × slant height
$= \pi(R + r)$ slant height

Slant height $= \sqrt{(3^2 + 9^2)} = \sqrt{(9 + 81)}$
$= \sqrt{90} = 9{\cdot}487$

Mid-circumference $= \pi(R + r)$
$= 15\pi = 47{\cdot}1$ in.

Surface area $= 47{\cdot}1 \times 9{\cdot}487$
$= \underline{446{\cdot}8 \text{ sq. in.}}$

EXAMPLE 8. *A cylindrical boiler* 16 *in. dia. and* 14 *in. deep is used to fill a quantity of cups. If the cups are in the form of right cone frustums and are* 3 *in. dia. at the top,* 2 *in. dia. at the base and* 3 *in. deep, calculate the number of cups that can be completely filled from this boiler.*

$$\begin{aligned}
\text{Volume of boiler} &= \pi r^2 h \\
&= \pi \times 8^2 \times 14 = 2813{\cdot}44 \text{ cu. in.} \\
\text{Volume of 1 cup} &= \tfrac{1}{3}\pi h(R^2 + r^2 + Rr) \\
&= \tfrac{1}{3} \times \pi \times 3(1\tfrac{1}{2}^2 + 1^2 + 1\tfrac{1}{2} \times 1) \\
&= \pi(4{\cdot}75) = 14{\cdot}91 \text{ cu. in.} \\
\text{No. of cups filled} &= \frac{\text{Volume of boiler}}{\text{Volume of 1 cup}} \\
&= \frac{2813{\cdot}44}{14{\cdot}91} = \underline{188}
\end{aligned}$$

EXAMPLE 9. *Find the capacity in gallons of an oval tapered vessel* 16 *in. by* 12 *in. at the bottom and* 22 *in. by* 18 *in. at the top and* 12 *in. deep. Also calculate the weight of the vessel, including the bottom, if made from metal weighing* 5 *lb per sq. ft* (*Fig.* 48).

$$\begin{aligned}
\text{Volume} &= [A + a + (4 \times \text{mid-area})]\,\frac{h}{6} \\
\text{Area } A &= \pi \times 11 \times 9 = 310{\cdot}86 \text{ sq. ins.} \\
\text{Area } a &= \pi \times 8 \times 6 = 150{\cdot}72 \text{ sq. ins.} \\
\text{Mid area} &= \pi \times 9{\cdot}5 \times 7{\cdot}5 \\
&= 224 \text{ sq. in.} \\
\therefore \text{Volume} &= \tfrac{12}{6}\,[310{\cdot}86 + 150{\cdot}72 + (4 \times 224)] \\
&= 2715{\cdot}16 \text{ cu. in.}
\end{aligned}$$

$$\text{Capacity} = \frac{2715{\cdot}16}{277{\cdot}4} = 9{\cdot}786 \text{ gallons (approx.)}$$

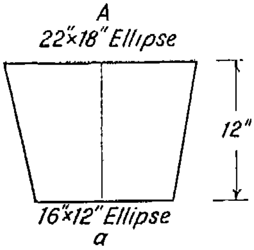


FIG. 48

$$\begin{aligned}
\text{Curved surface area} &= \text{mid-perimeter} \times \text{slant height} \\
\text{Slant height} &= \sqrt{(12^2 + 3^2)} = \sqrt{153} = 12{\cdot}37 \text{ in.} \\
\text{Surface area} &= \frac{\pi}{2}\,[(R_1 + R_2) + (r_1 + r_2)] \times 12{\cdot}37 \\
&= \pi(9{\cdot}5 + 7{\cdot}5) \times 12{\cdot}37 \\
&= \pi \times 17 \times 12{\cdot}37 = 660{\cdot}31 \text{ sq. in.} \\
\text{Area of base} &= \pi(8 + 6) = 43{\cdot}96 \text{ sq. in.} \\
\text{Weight of vessel} &= \frac{704{\cdot}27}{144} \times 5 = 24{\cdot}45 \text{ lb} \\
\text{Weight of water in tank} &= 10 \times 10 = 100 \text{ lb} \\
\text{Total weight} &= 100 + 24{\cdot}45 = \underline{124{\cdot}45 \text{ lb}}
\end{aligned}$$

Exercise 12

1. Calculate the volume and curved surface area of a right cone 6½ in. dia. at the base and 7½ in. vertical height.
2. A paraffin-oil can has a cylindrical body and a conical top. The diameter is 8½ in. and the depth of the body is 9½ in., the conical top, which may be taken as a complete cone, has a slant height of 5 in. Calculate (*a*) the capacity in gallons of the can, (*b*) the weight of paraffin the can will hold when full. (Sp. gr. of paraffin = 0·73.)
3. A boiler, in the form of a cone frustum, is 30 in. dia. at the top, 18 in. dia. at the base, and the slant height is 10 in. Calculate (*a*) the capacity of the boiler in gallons, (*b*) the weight of water in the boiler when full.
4. 100 square-based pyramids are to be made from copper weighing 4·3 lb/sq. ft. Calculate the total weight of the pyramids if the base of each pyramid is open, the side measures 5·7 in., the vertical height is 9½ in., and 7% additional metal is used for laps and seams.
5. A vessel, in the form of a cone frustum, is 2 ft diameter at the top, 1 ft diameter at the base and 1 ft 6 in. vertical height. If a small cylindrical vessel 4 in. diameter and 5 in. deep is used to empty the larger vessel, how many times can it be filled from the liquid contained in the larger vessel which is 80% full?
6. A piece of lead, in the form of a cube, with 2-in. sides is melted down and cast into a mould in the form of a right cone. If the vertical height of the cone is 3 in., calculate the base diameter.
7. Calculate the volume of a sphere with an internal diameter of 6·27 in.
8. A spherical float is 12 in. in diameter and weighs 3 lb. If the float is made from metal weighing 0·32 lb/cu. in., calculate the thickness of the metal used for making the float.
9. A boiler consists of a hemispherical base with a vertical cylindrical upper portion. If the boiler diameter is 1 ft 9 in. and the overall depth is 2 ft 6 in., calculate the capacity of the boiler in gallons.
10. A feed hopper, in the form of a frustum of a square pyramid, has sides 2 ft long at the top and 3 ft 9 in. long at the bottom. If the vertical height of the hopper is 21 in., calculate its volume in cubic feet.

Graphs

A great deal of information can be more readily understood if it is presented in pictorial form instead of as a table of figures and to this end graphs are used.

Consider the following statistics showing the output of parts from a certain firm in six monthly-periods:

	1	2	3	4	5	6	7	8	9	10
	1953		1954		1955		1956		1957	
Period	Jan.	Jul.	Jan.	Jul.	Jan.	Jul.	Jan.	Jul.	Jan.	Jul.
No. of units	190	196	212	216	238	268	283	311	371	382

The table shows the output rising but does not readily show the *rate* of rise.

Now look at the graph drawn from these figures, Fig. 49.

Here we can see immediately that there was a steady but gradual rise for the first four periods and then the rate of rise increased considerably. We get from the graph a better understanding of the trend of output than we get from the table of figures.

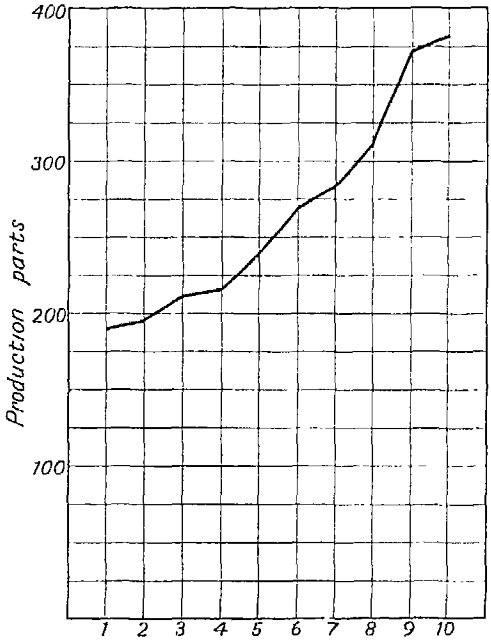


FIG. 49

Another useful kind of graph is one that shows the connection between two related values such as: hours worked: wages, degrees Centigrade: degrees Fahrenheit, inches: millimetres, etc.

Plotting a Graph

The drawing of a graph from known data is called ‘ plotting the graph ’. Graph paper is used which is marked off in squares, usually in inches and tenths of an inch.

Two lines are drawn, one along the bottom called the horizontal axis and the second down the left-hand side called the vertical axis. These must be placed to allow the units and scale to be inserted as shown in Fig. 49.

Scales are then chosen so that the whole of the data required from the graph can be covered on each axis. For ease of plotting and reading, the scales should be in tens or multiples of ten. Since the accuracy of the graph is governed by its size the scales should be as large as possible.

To plot the graph of inches : millimetres from the following values:

inch	·25	·5	·75	1
mm	6·35	12·7	19·05	25·4

Mark off the horizontal and vertical axes.

If inches are to be plotted on the horizontal axis the range is from zero to 1 in. by steps of ·25 in., then let each inch on the graph paper represent ·25 in.

The mm. range is from zero to 25·4 let each inch of graph paper represent 5 mm.

Insert the scale values and units.

·25 in. corresponds to 6·35 mm. Therefore starting from ·25 in. on the horizontal axis move up this line until the horizontal line from 6·35 mm. meets it; this is the first point to be marked.

Repeat for the other values from the table.

Draw the line through the points to finish the graph.

Note that neither axis scale was wholly used up because any larger scale would have produced awkward scale values.

From this graph any inch or mm. size within the graph values could be converted to the corresponding unit by the same method as was used for plotting,

e.g., ·375 in. is equivalent to 9·52 mm.

15 mm. is equivalent to ·591 in.

Since the relationship inches : mm. is constant, the range of the graph could be extended by extending the line and scales to the limits of the paper as shown in Fig. 50. The range of the first graph, Fig. 49, showing time and output could not be extended except by guesswork, since the output is not governed by time alone but by a host of other factors; material supply, orders in hand, men available, etc.

Many connected factors do not have this simple, direct relationship, that is, the graph is not a straight line but a curve. This type of graph is made by plotting the points and joining them by a smooth curve.

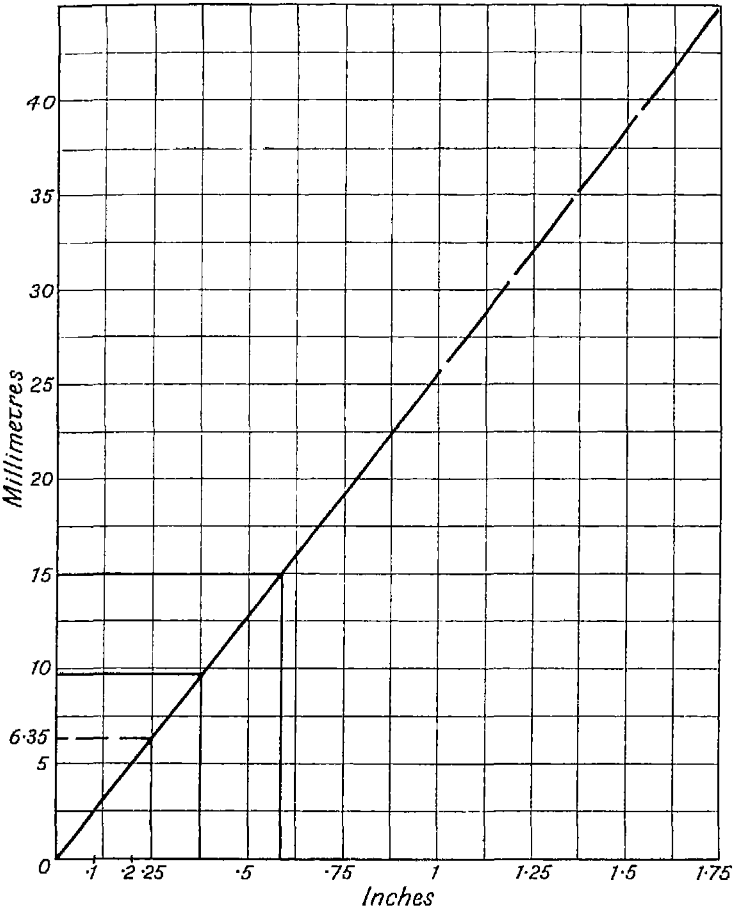


FIG. 50

EXAMPLE 1. *Plot the graph of the diameter and area of a circle from the following values:*

Dia. inches	1	2	3	4	5	6	7
Area square inches	·7854	3·141	7·068	12·56	19·63	28·27	38·48

From your graph find (a) the area of a circle of 2·5 *in. dia., (b) the diameter of a circle of area* 16·8 *sq. in.*

The plotted graph is shown in Fig. 51. From the graph the area of a 2·5 in. diameter circle is 4·908 sq. in. and the diameter of a circle of area 16·8 sq. in. is 4·625 in.

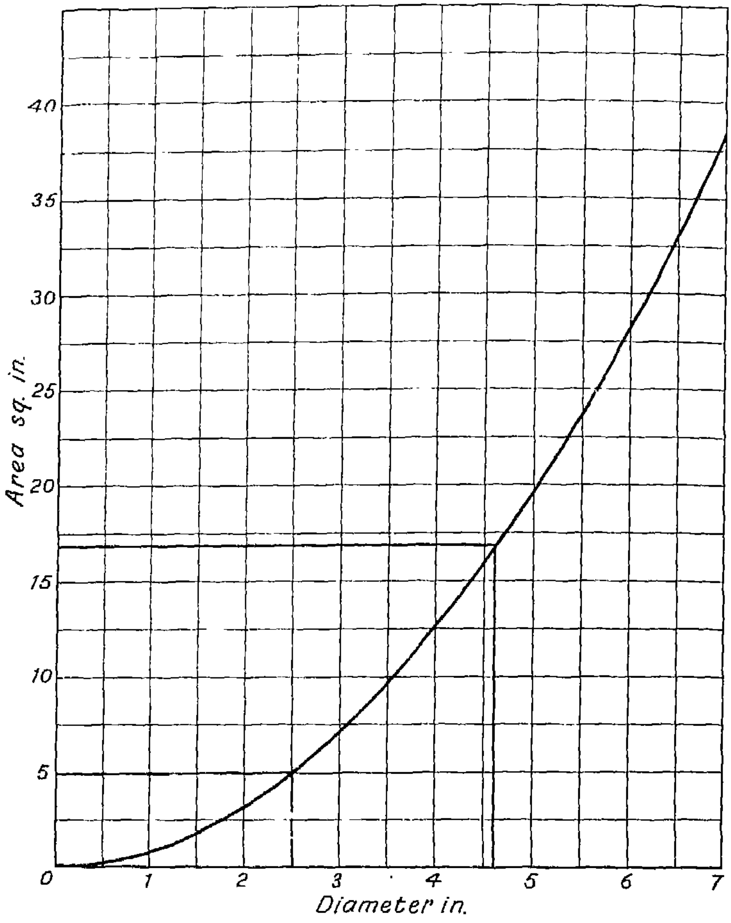


FIG. 51

Further Notes on Graphs

Although the values in a graph are usually plotted from zero on both axes it is sometimes convenient to start from a greater value on *one or both axes*. For example, if one range of values was from 175 up to 250 in eight steps a larger scale could be used if these values were started at 170 instead of zero. Of course this can be done only if the lower values are not required.

The first problem in plotting a graph is to decide which axis to use for the two quantities. The rule is that the horizontal axis is called the *independent* axis while the vertical is called the *dependent* axis. In other words, if one quantity controls the other the controlling one is independent and is plotted on the horizontal axis.

The area of a circle depends on its diameter, the diameter is independent and goes on the horizontal axis.

The strength of a rivet depends on its diameter, so the diameter is plotted on the horizontal axis.

In some graphs either axis could be used, for example, the graph of millimetres and inches.

If the graph to be plotted is known to be a straight line two points would be sufficient to locate the line; however, one more point should be fixed as a check on the other two points.

In a straight line graph, if x is a value on the horizontal axis and y is the corresponding vertical value, then the value of y is obtained by multiplying the x value by a constant number; call this number m.

$$y = m \times x$$

$$\text{or sometimes } y = m \times x + c$$

c is another constant number which is added to the product $m \times x$ and is the amount up the y axis where the plotted line crosses the y axis.

No powers or roots are involved in a straight line graph.

Circumference, $C = \pi d$. mm. $= 25{\cdot}4 \times$ inches, both these will be straight line graphs.

Circle area, $A = \pi r^2$. Length of side of a square. $L = \sqrt{\text{area}}$. These will be curves, since a square and a square root are involved.

Exercise 12

1. The length of wire in a one hundredweight coil for different diameters of wire is given below:

Wire dia. in.	·3	·276	·252	·232	·212	·192
Feet per cwt.	480	570	684	807	966	1179

Plot a graph from these data.

2. During a tensile test of a mild steel test piece the specimen was pulled up to a series of loads and the corresponding stretch of the part was noted. From the results given below plot the graph of load : extension.

Load, thousand pounds	1	3	5	7	9	11	12	13
Extension, thousandths of an inch	·66	2	3·3	4·7	6	7·3	8	8·7

(*Note*, plot load on the vertical axis.)

3. A tank holds 130 gall. of water. When the tap is turned on full the volume falls as shown.

   (*a*) How long will it be before the tank is half empty?

   (*b*) How many gall. will there be after 18 sec.?

t sec.	0	5	10	15	20	25	30	35
v. gall.	130	117	104	92	78	66	53	40

4. The wiring allowance for thin sheet edges is calculated from, allowance = 2·5 d, where d is the wire diameter. Fill in the table shown for the wire diameters given and plot the graph.

   From the graph find the allowance for wire of ·212 in. diameter.

Wire dia. in.	·276	·232	·192	·160	·128	·104	·080	·064
Allowance	·69	·58						

5. The circumference of a circle is found from: circumference = πd, where π is 3·14 and d is the circle diameter. Calculate the circumferences of circles from 1 in. to 6 in. in steps of one inch, tabulate these values and from them draw the graph of diameter : circumference.
6. The cross-section area of a square pipe is calculated from: area = l^2, where l is the length of side. Calculate the areas of pipes from 1 in. to 10 in. side length, tabulate these values and from them draw the graph of side length : area.
7. The following table gives the weight of mild steel sheet per square foot for different standard wire gauges. Plot the graph of gauge and weight.

Imperial S.W.G.	10	12	14	16	18	20	22	24
Lb/sq. ft	5·27	4·28	3·30	2·64	1·98	1·47	1·15	0·905

   From the graph find the weight per square foot of 13 and 17 S.W.G. sheet.

Miscellaneous Worked Examples

1. *A water tank is to be made with the following internal dimensions:* 11·57 in. *by* 9·31 *in. by* 5·66 *in. deep. The rule used for measuring the plates is graduated in steps of* $\frac{1}{8}$ *in. only, and the plates are cut to the nearest graduation above the given dimensions. Calculate the percentage error in the volume of the tank produced.*

$$\text{Now } 11{\cdot}57 \simeq 11\tfrac{5}{8}$$
$$\text{and } 9{\cdot}31 \simeq 9\tfrac{3}{8}$$
$$\text{and } 5{\cdot}66 \simeq 5\tfrac{3}{4}$$

$$\begin{aligned}
\therefore\ \text{Volume of tank required} &= 11{\cdot}57 \times 9{\cdot}31 \times 5{\cdot}66 \\
&= 609{\cdot}5 \text{ cu. in.} \\
\text{Volume of tank produced} &= 11\tfrac{5}{8} \times 9\tfrac{3}{8} \times 5\tfrac{3}{4} \\
&= 626\tfrac{6}{10} = 626{\cdot}6 \text{ cu. in.} \\
\text{Error} &= 17{\cdot}1 \text{ cu. in.} \\
\text{Percentage error} &= \frac{17{\cdot}1}{609{\cdot}5} \times 100 \\
&= \underline{2{\cdot}8 \text{ per cent.}}
\end{aligned}$$

2. *A cylindrical tank of 2 ft 6 in. diameter and 4 ft vertical height is half filled with water and drained off into a rectangular tank 3 ft 6 in. long, 2 ft wide and 1 ft 6 in. deep. Calculate the height of the water in the rectangular tank when the cylindrical tank is empty.* (C.G.L.I.)

Volume of water in cylindrical tank

$$= \frac{\pi d^2}{4}h = \frac{\pi \times 2{\cdot}5^2 \times 2}{4}$$

$= 9{\cdot}812$ cu. ft.

End area of rectangular tank $= 3\frac{1}{2} \times 2 = 7$ sq. ft

For equal volumes of water then, $9{\cdot}812 = 7 \times$ depth

$\therefore$ Depth $= \dfrac{9{\cdot}812}{7} = 1{\cdot}401$ ft

$= 16{\cdot}812$ in.

say 16·8 in.

3. *A tank 14 in. long, 11 in. wide and 6·5 in. deep is filled to a depth of 6·1 in. with dilute acid. If a 1-pint measure is filled seven times from the tank, calculate the depth of the acid remaining.*

Now, 1 gallon $= 277{\cdot}4$ cu. in.

$\therefore$ 7 pints $= 277{\cdot}4 \times \frac{7}{8}$

$= 242{\cdot}725$ cu. in.

End area of tank $= 14 \times 11 = 154$ sq. in.

Volume = end area × depth

$\therefore$ $242{\cdot}725 = 154 \times$ depth

Depth $= \dfrac{242{\cdot}725}{154} = 1{\cdot}576$ in.

Depth of acid remaining $= 6{\cdot}1 - 1{\cdot}576$

$= 4{\cdot}524$ in.

say 4·5 in.

4. *A lead-lined tank is 6 ft long, 2 ft 6 in. wide, 2 ft deep and has a turnover round the top of 6 in. Calculate (a) the weight of lead required to line the tank with 10 lb lead, (b) the volume of muriatic acid required to give a 5% pickle filling the tank to within 8 in. of the top* (Fig. 52).

(*a*) Area of base $= 6 \times 2{\cdot}5 = 15$ sq. ft.

Area of sides $= 2(6 + 2{\cdot}5) \times 2 = 34$ sq. ft.

Area of top $= (7 \times 3{\cdot}5) - (6 \times 2{\cdot}5)$

$= 24{\cdot}5 - 15 = 9{\cdot}5$ sq. ft.

Total area $= 15 + 34 + 9{\cdot}5 = 58{\cdot}5$ sq. ft

Weight of lead $= 58{\cdot}5 \times 10 = 585$ lb.

Volume of liquid in tank $= 6 \times 2{\cdot}5 \times 1{\cdot}33$

$= 19{\cdot}95$ cu. ft $= 20$ cu. ft (approx.)

Volume of acid $= \dfrac{20 \times 5}{100} = 1$ cu. ft.

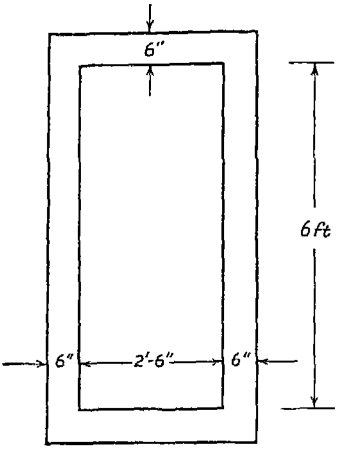


FIG. 52

5. *A tank* 20 *in. square and a circular tank* 20 *in. diameter each have a depth of* 15 *in. and are full of water; which holds the greater weight of water and by how much? Compare the perimeter of each tank.* (*Take* 1 *cu. ft of water as* 62·5 *lb.*)

$$\text{Volume of square tank} = 1\tfrac{2}{3} \times 1\tfrac{2}{3} \times 1\tfrac{1}{4}$$
$$= \tfrac{5}{3} \times \tfrac{5}{3} \times \tfrac{5}{4} = \tfrac{125}{36}$$
$$= 3\tfrac{17}{36} \text{ cu. ft.}$$
$$\text{Volume of circular tank} = \frac{\pi d^2}{4} \times l$$
$$= \frac{22 \times 1\tfrac{2}{3} \times 1\tfrac{2}{3} \times 1\tfrac{1}{4}}{7 \times 4}$$
$$= \frac{22 \times 5 \times 5 \times 5}{7 \times 4 \times 3 \times 3 \times 4} = 2\tfrac{367}{504} \text{ cu. ft.}$$

$\therefore$ The square tank holds the greater weight of water.

$$\text{Difference in volume} = 3\tfrac{17}{36} - 2\tfrac{367}{504}$$
$$= 3{\cdot}472 - 2{\cdot}728$$
$$= 0{\cdot}744 \text{ cu. ft}$$
$$\text{Difference in weight} = 0{\cdot}744 \times 62{\cdot}5$$
$$= 46{\cdot}5 \text{ lb}$$
$$\text{Perimeter of square tank} = 4 \times 20 = 80 \text{ in.}$$
$$\text{Perimeter of circular tank} = \pi \times 20 = 62{\cdot}8 \text{ in.}$$
$$\text{Ratio of perimeter} = 80 : 62{\cdot}8$$
$$= \underline{1{\cdot}273 : 1.}$$

6. *A rectangular tank* 24 *in. long by* 18 *in. wide by* 16 *in. deep is filled by emptying buckets of water into it. If the bucket is* 10 *in. diameter at the top,* 8 *in. diameter at the bottom and* 10 *in. depth, calculate* (*a*) *how many buckets full of water will*

be required to fill the tank, (b) how deep the water will be in the tank when 8 buckets of water have been emptied into it.

Volume of tank $= 24 \times 18 \times 16$
$= 6912$ cu. in.
Volume of bucket $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$
$= \frac{1}{3}\pi \times 10[5^2 + 4^2 + (5 \times 4)]$
$= \frac{1}{3} \times \pi \times 10(25 + 16 + 20)$
$= 638{\cdot}46$ cu. in.
Total no. of buckets required $= \dfrac{\text{Volume of tank}}{\text{Volume of one bucket}}$
$= \dfrac{6912}{638{\cdot}46} = 10{\cdot}82$
Volume of 8 buckets $= 638{\cdot}46 \times 8$
$= 5107{\cdot}68$ cu. in.
End area of tank $= 24 \times 18$
$= 432$ sq. in
Depth of water in tank $= \dfrac{5107{\cdot}68}{432}$
$= \underline{11{\cdot}82 \text{ in.}}$

7. *A fuel oil tank 4 ft deep has horizontal elliptical ends of 3 ft and 2 ft major and minor radii. The oil is drained off at frequent intervals to fill a quantity of cylindrical containers of 10 in. diameter and 14 in. depth. During this process 70 gallons of oil are taken from the elliptical tank. Calculate (a) the minimum number of cylindrical containers required, (b) the area of metal inside the elliptical tank released from contact with the oil. (277·4 cu. in. = 1 gallon.)*

Volume of oil taken from tank $= 277{\cdot}4 \times 70$
$= 19418$ cu. in.
Volume of one cylinder $= \pi r^2 h = \pi \times 5^2 \times 14$
$= 1099$ cu. in.
No. of containers required $= \dfrac{19418}{1099} = 17{\cdot}6$
$\therefore$ $\underline{\text{18 containers are required.}}$

End area of fuel tank $= \pi Rr$
$= \pi \times 18 \times 12$
$= 678{\cdot}24$ sq. in.
Depth of sink $= \dfrac{\text{volume extracted}}{\text{end area}}$
$= \dfrac{19418}{678{\cdot}24} = 28{\cdot}63$ in.
Perimeter of fuel tank $= \pi(R + r) = \pi(18 + 12)$
$= 94{\cdot}2$ in.
Area of metal released $= 94{\cdot}2 \times 28{\cdot}63$
$= \underline{2696{\cdot}94 \text{ sq. in.}}$

8. *A main air duct of 2 ft 6 in. diameter and 100 ft long is to be replaced with one of rectangular section of the same effective area. If the rectangle is twice as long as it is wide, calculate the extra weight of material used, assuming that in both ducts the metal weighs $2\frac{1}{2}$ lb/sq. ft.*

Curved surface area of circular duct $= \pi dl$

$= \pi \times 2{\cdot}5 \times 100$

$= 785$ sq. ft

Weight of metal $= 785 \times 2{\cdot}5$

$= 1962{\cdot}5$ lb

Effective area of ducts $= \dfrac{\pi d^2}{4} = \dfrac{\pi \times 2{\cdot}5^2}{4}$

$= 4{\cdot}908$ sq. ft

Area of rectangular duct $= L \times B$

$\therefore$ Let $x = B$ and $2x = L$

$\therefore$ $4{\cdot}908 = 2x \times x$

$4{\cdot}908 = 2x^2$

From which, $x = \sqrt{\dfrac{4{\cdot}908}{2}}$

$x = 1{\cdot}566$ ft

$2x = 3{\cdot}132$ ft

Then, length of rectangular duct $= 3{\cdot}132$ ft

Width of rectangular duct $= 1{\cdot}566$ ft

$\therefore$ Perimeter $= 2(3{\cdot}132 + 1{\cdot}566)$

$= 9{\cdot}396$ ft

Surface area $= 9{\cdot}396 \times 100$

$= 939{\cdot}6$ sq. ft

Weight of metal $= 939{\cdot}6 \times 2{\cdot}5$

$= 2349$ lb

Extra weight $= 2349 - 1962{\cdot}5$

$= \underline{386{\cdot}5 \text{ lb.}}$

9. *The formula required to find the area of metal* A *in a boiling pan with a hemispherical base and a vertical cylindrical upper portion is* $A = 2\pi r^2 + 2\pi rh$. *Where* r = *radius of cylinder and hemisphere and* h = *height of the vertical cylindrical sides. Transpose this formula to give* h *in terms of* A, r *and* π *and hence find* h *when* A = 1300 *sq. in and* r = 10 *in.*

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 2\pi r(r + h)$$

$$\frac{A}{2\pi r} = r + h$$

$$\therefore\ h = \frac{A}{2\pi r} - r$$

Substituting values of A and r

$$\begin{aligned} h &= \frac{1300}{2\pi \times 10} - 10 \\ &= \frac{1300}{62{\cdot}8} - 10 \\ &= 20{\cdot}69 - 10 \\ &= \underline{10{\cdot}69 \text{ in.}} \end{aligned}$$

10 *The total cost of producing a quantity of articles was £200. Of this the material accounted for 23 %, and the labour 34 %, whilst the remainder was due to overhead charges. At a later date a similar order was received and by this time the price of material had risen 12 % and the cost of labour 15 %. If the overhead charges for both orders remained unaltered, calculate the cost of the second order.*

1*st Order*

$$\text{Material} = \overset{2}{\cancel{200}} \times \frac{23}{\cancel{100}} = \text{£}46$$

$$\text{Labour} = \frac{\overset{2}{\cancel{200}} \times 34}{\cancel{100}} = \text{£}68$$

$$\text{Overheads} = 200 - (68 + 46) = \text{£}86$$

2*nd Order*

$$\text{Material} = 46 \times \frac{112}{100} = \text{£}51{\cdot}52$$

$$\text{Labour} = 68 \times \frac{115}{100} = \text{£}78{\cdot}2$$

$$\text{Overheads} = \underline{\text{£}86}$$

$$\text{Cost of second order} = \text{£}215{\cdot}72 = \underline{\text{£}215\ 14s.\ 5d.}$$

11. *A hopper in the form of a frustum of a square pyramid is 6 ft square at the top and 3 ft square at the bottom. If its vertical height is 4 ft 6 in., calculate its capacity in cubic feet.*

Let A = top area

a = bottom area

h = vertical height

$$\begin{aligned} \text{Then, Volume} &= \tfrac{1}{3}h[A + a + \sqrt{(Aa)}] \\ &= \frac{4{\cdot}5}{3}[6^2 + 3^2 + \sqrt{(6^2 \times 3^2)}] \\ &= 1{\cdot}5[36 + 9 + (6 \times 3)] \quad \text{as } \sqrt{6^2} = 6 \text{ and } \sqrt{3^2} = 3 \\ &= 1{\cdot}5 \times 63 = \underline{94{\cdot}5 \text{ cubic feet.}} \end{aligned}$$

12. *100 discs 13·5 in. in diameter are cut from sheets of monel metal 0·048 in. thick. Monel metal is composed of 67% nickel, 30% copper, 1½% iron, and 1½% manganese. Calculate the weight of the 100 discs. (Specific gravity of nickel = 8·68, copper 8·8, iron 7·8, manganese 8·0. 1 cu. in. of water weighs 0·036 lb.)*

$$\begin{aligned}
\text{Specific gravity of monel metal} &= 8{\cdot}68 \times 0{\cdot}67 = 5{\cdot}816 \text{ nickel}\\
&= 8{\cdot}8 \times 0{\cdot}3 = 2{\cdot}64 \text{ copper}\\
&= 7{\cdot}8 \times 0{\cdot}015 = 0{\cdot}117 \text{ iron}\\
&= 8 \times 0{\cdot}015 = 0{\cdot}12 \text{ manganese}\\
\text{Total} &= 8{\cdot}693\\
&= 8{\cdot}7 \text{ approx.}\\
\text{Wt./cu. in. of monel metal} &= 8{\cdot}7 \times 0{\cdot}036\\
&= 0{\cdot}3132 \text{ lb.}\\
\text{Volume of discs} &= \frac{\pi d^2}{4} \times \text{thickness} \times 100\\
&= \frac{\pi \times 13{\cdot}5^2 \times 0{\cdot}048 \times 100}{4}\\
&= 686{\cdot}7 \text{ cu. in.}\\
\text{Wt. of discs} &= 687{\cdot}1 \times 0{\cdot}3132 = \underline{214{\cdot}9 \text{ lb.}}
\end{aligned}$$

13. *A 1-gallon cylindrical measure of 5½ in. diameter is to be graduated by means of beads, each division holding one pint. Calculate the depth required for each bead, and the overall depth of the measure.*

$$\begin{aligned}
1 \text{ gallon} &= 277{\cdot}4 \text{ cu. in.}\\
1 \text{ pint} &= \frac{277{\cdot}4}{8} = 34{\cdot}67 \text{ cu. in.}\\
\text{End area of measure} &= \frac{\pi d^2}{4}\\
&= \frac{\pi \times 5\tfrac{1}{2}^2}{4}\\
&= 23{\cdot}75 \text{ sq. in.}\\
\text{Distance between each bead} &= \frac{34{\cdot}67}{23{\cdot}75}\\
&= 1{\cdot}46 \text{ in.}\\
\text{Overall depth of measure} &= 1{\cdot}46 \times 8\\
&= \underline{11{\cdot}68 \text{ in.}}
\end{aligned}$$

14. *Two cylindrical tanks each 20 in. in diameter and 24 in. deep are to be replaced by a single steam-heated rectangular tank of equal cubic capacity. The length of the tank is 36 in. and the width 18 in. Allowing in each case 10% for steam coils, calculate (a) the required depth of the tank, (b) its capacity in gallons. (277·4 cu. in. to 1 gallon.)*

(*a*) Volume of cylindrical tanks $= \pi r^2 h \times 2$

$= \pi \times 10^2 \times 24 \times 2 = 15{,}072$ cu. in.

$+ 10\% = 16{,}579{\cdot}2$ cu. in.

End area of rectangular tank $= 36 \times 18$

$= 648$ sq. in.

$\therefore$ Depth of rectangular tank $= \dfrac{16579{\cdot}2}{648}$

$= \underline{25{\cdot}58 \text{ in.}}$

(*b*) Capacity $= \dfrac{15072}{277{\cdot}4}$

$= \underline{54{\cdot}33 \text{ gallons.}}$

15. *A piece of sheet metal in the form of a sector of a circle of* 10 *in. radius, containing an angle of* 135 *degrees, is shaped to form a hollow cone. Calculate* (*a*) *the volume of the cone,* (*b*) *its surface area.*

$$\theta^\circ = \frac{180 \times D}{R}$$

Where $D =$ base diameter

$R =$ slant height or pattern radius

$$\therefore \quad 135 = \frac{180 \times D}{10}$$

From which, $D = \dfrac{135 \times 10}{180}$

$= 7{\cdot}5$ in.

Vertical height of cone $= \sqrt{(10^2 - 3{\cdot}75^2)}$

$= \sqrt{(100 - 14{\cdot}06)}$

$= \sqrt{85{\cdot}94}$

$= 9{\cdot}27$ in.

(*a*) Volume of cone $=$ area of base $\times \frac{1}{3}$ vertical height

$$= \frac{\pi \times 7{\cdot}5^2}{4} \times \frac{9{\cdot}27}{3}$$

$= \underline{136{\cdot}5 \text{ cu. in.}}$

(*b*) Surface area $=$ area of sector

$$= \pi r^2 \times \frac{135}{360} = \frac{\pi \times 10^2 \times 135}{360}$$

$= \underline{117{\cdot}8 \text{ sq. in.}}$

16. *A fuel tank, with a closed horizontal top, is made of sheet metal and is* 16 *in. long by* 12 *in. wide. It is* 15 *in. deep at one end sloping to* 30 *in. at the other end, measured lengthwise. Assuming that the material is* $\frac{1}{16}$ *in. plate, and allowing* 5% *for seams, calculate* (*a*) *the capacity of the tank in gallons when full,* (*b*) *the weight of metal used.* (1 *cu. in. of metal weighs* 0·3 *lb.*) (Fig. 53.)

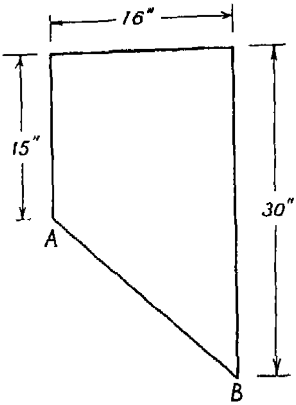


FIG. 53

$$\text{Average depth} = \frac{30 + 15}{2}$$

$$= 22{\cdot}5 \text{ in.}$$

$$\text{Volume of tank} = \text{end area} \times \text{average depth}$$

$$= 16 \times 12 \times 22{\cdot}5$$

$$= 4320 \text{ cu. in.}$$

$$\text{Capacity} = \frac{277{\cdot}4}{4320} = \underline{15{\cdot}57 \text{ gallons.}}$$

Area of metal in tank:

$$\text{Area of horizontal top} = 16 \times 12 = 192 \text{ sq. in.}$$

Area of inclined base:

$$\text{Now true length AB} = \sqrt{(15^2 + 16^2)}$$

$$= \sqrt{(225 + 256)}$$

$$= \sqrt{481}$$

$$= 21{\cdot}9 \text{ in.}$$

$$\therefore \quad \text{Area of inclined base} = 21{\cdot}9 \times 12$$

$$= 262{\cdot}8 \text{ sq. in.}$$

$$\text{Area of sides} = 30 \times 12 = 360 \text{ sq. in.}$$

$$15 \times 12 = 180 \text{ sq. in.}$$

$$2 + \left(\frac{15 + 30}{2}\right) \times 16 = 720 \text{ sq. in.}$$

$$\text{Total area of metal in tank} = 192 + 262{\cdot}8 + 360 + 180 + 720$$

$$= 1714{\cdot}8 \text{ sq. in.}$$

$$\text{Volume of metal in tank} = 1714{\cdot}8 \times \tfrac{1}{16}$$

$$= 108{\cdot}1 \text{ cu. in.}$$

$$\text{Allowance for seams} = 108{\cdot}1 \times \frac{105}{100}$$

$$= 113{\cdot}505 \text{ cu. in.}$$

$$\text{Weight of metal in tank} = 113{\cdot}505 \times 0{\cdot}3$$

$$= 34{\cdot}0515 \text{ lb} \quad \underline{\text{say 34 lb.}}$$

17. *A closed fuel tank made of sheet metal* 0·02 *in. thick is* 30 *in. deep. The tank in plan is a rectangle* 16 *in. long and* 12 *in. wide, with* 3 *in. radius corners. The top and bottom are flat. Calculate* (*a*) *the capacity of the tank in gallons*, (*b*) *the weight of the empty tank after allowing* 10% *for seams, etc.* (1 *cu. in. of metal* = 0·3 *lb.*) (Fig. 54.)

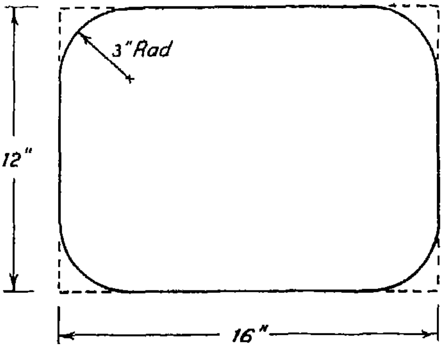


FIG. 54

End area of tank = area of rectangle − area of dotted portion

Area of rectangle $= 12 \times 16 = 192$ sq. in.

If the four corners were put together they would make up a 6 in. side square and a 6 in. dia. circle.

∴ Area of dotted portions $= 6^2 - \pi \times 3^2$
$= 36 - 9\pi = 36 - 28{\cdot}26$
$= 7{\cdot}74$ sq. in.

∴ End area of tank $= 192 - 7{\cdot}74$
$= 184{\cdot}26$ sq. in.

Volume of tank = end area × depth
$= 184{\cdot}26 \times 30$
$= 5527{\cdot}8$ cu. in.

Capacity of tank $= \dfrac{5527{\cdot}8}{277{\cdot}4}$
$= 19{\cdot}93$ gallons.

Perimeter of tank = circumference of a 6 in. diameter circle and four straight sides
$= 6\pi + 2(10 + 6)$
$= 18{\cdot}84 + 32 = 50{\cdot}84$ in.

Area of sides $= 50{\cdot}84 \times 30 = 1525{\cdot}2$ sq. in.

Area of ends $= 2 \times 184{\cdot}26 = 368{\cdot}52$ sq. in.

Total area of metal in tank $= 1525{\cdot}2 + 368{\cdot}52$
$= 1893{\cdot}72$ sq. in.

$$\begin{aligned}
\text{Volume of metal in tank} &= 1893{\cdot}72 \times 0{\cdot}02 \\
&= 37{\cdot}87 \text{ cu. in.} \\
\text{Allowance for seams, etc.} &= 37{\cdot}87 \times \frac{110}{100} \\
&= 41{\cdot}657 \text{ cu. in.} \\
\text{Weight of empty tank} &= 41{\cdot}657 \times 0{\cdot}3 \\
&= 12{\cdot}397 \text{ lb} \\
&\underline{\text{say } 12{\cdot}4 \text{ lb.}}
\end{aligned}$$

Miscellaneous Exercises

1. A cylindrical drum is to be made 3 ft long to fit into a recess. If it is to have a volume of 10 cubic ft calculate its diameter.
2. Calculate the area of metal required to make a closed cylindrical container 10 in. diameter and 8 in. high. (Neglect laps.)
3. A cylindrical tank is 9 ft diameter and 14 ft high. If it is filled with water to within 2 ft of the top how many gallons does it hold? (1 cubic ft holds 6·25 gallons.)
4. A tin is made to hold one pound of liquid and is 4 in. in diameter and 8 in. high. It is decided to make ¼ lb tins of 3 in. diameter. Calculate the required height of the ¼ lb tins.
5. 1000 cylindrical tins closed at one end are to be made and they are to be 4·5 in. in diameter and 6 in. high. Calculate the total area of metal in sq. ft required if 35% is to be allowed for laps and scrap.
6. A hollow cone pattern is cut on an 8 in. radius and contains an angle of 120°. Calculate the diameter of the made-up cone. (Neglect laps.)
7. A sheet metal cone is 12 in. in diameter and its vertical height is 8 in. Calculate (*a*) its volume, (*b*) the area of metal required to make it.
8. A certain grease is kept in a container ·9 in. in diameter and 3½ in. long. It is used by pressing it out of a nozzle ·18 in. diameter. If it is used twice a day and each time a length of ·35 in. is squeezed out, how many days will the grease last?
9. The cost of a certain article is made up as follows: material 1/6*d*., labour 1/-, overheads 6*d*., and the manufacturer sells it at 6/-. The labour costs rise 100% and the manufacturer raises his price by 50%. What is the change in the manufacturer's profit per cent?
10. A manufacturer reckons that his costs are made up of material, labour and overheads in the proportion 5 : 3 : 2. These costs rise by 45%, 60% and 30% respectively. What is the percentage increase in the total costs?

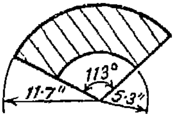


FIG. 55

11. Calculate the area of the part annulus shown shaded in Fig. 55.
12. A cylindrical water tank is 3 ft in diameter and 6 ft high. The tank is half filled with water which is transferred to a similar tank 2½ ft in diameter and 6 ft high. Calculate the height of the water in the smaller diameter tank.
13. A cylindrical tank is 12 in. high and when it is filled to 80% of its capacity contains 62·5 lb of water. Calculate (*a*) the diameter of the tank, (*b*) the weight of metal required to make the tank if 5% is allowed for laps and seams. (1 cu. in. of water weighs ·036 lb., 1 sq. ft of metal weighs 7·5 lb.)

14. A circular duct of 16 in. diameter has two circular branch pipes one of which is to be 8 in. in diameter. (*a*) Calculate the diameter of the second branch if the two pipes have to carry a volume of air equal to that of the main duct. (*b*) Show graphically how to obtain the diameters of the two branch pipes in terms of the diameter of the main pipe.
15. A junction piece is made up of a main circular pipe and two branches, one circular and the other of square section. If the main pipe diameter is 14 in., calculate the dimensions of the branches if they have to have equal effective areas.
16. Calculate the weight of metal and capacity of an elliptical section tank of 14 in. major axis and 12 in. minor axis and $18\frac{1}{2}$ in. deep if it is made of sheet metal weighing $4\frac{1}{2}$ lb per sq. ft. (Neglect laps or seams.) (1 cubic foot holds 6·25 gallons.)
17. A hopper is made in the form of the frustum of a square pyramid. The top length of side is 8 ft, the bottom length of side is 4 ft and the vertical height is 6 ft. Calculate the capacity of the hopper.

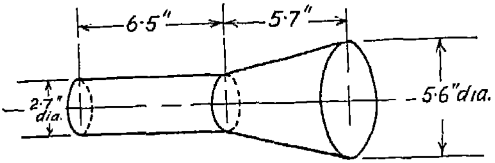


FIG. 56

18. A hopper is to be made in the form of the frustum of a cone. The top diameter is $4\frac{1}{2}$ ft, and the bottom diameter $3\frac{1}{2}$ ft, with the vertical height $2\frac{3}{4}$ ft. Calculate the area of metal required to make it if laps and seams are neglected.
19. A 24-in. diameter main pipe divides into 3 branches (2 circular and 1 rectangular). The rectangular branch is to take one half the available area, and its sides are in the ratio of 3 : 1. The circular pipes take up the remaining effective area and are in a ratio of 7 : 4. Calculate the dimensions of the three outlets.
20. A metal blank of 18 in. diameter is required to have a flange raised at the outer edge at right angles to the plane of the blank. An additional width of 4 in. is allowed all round the blank making a full diameter of 26 in. Assuming that no thinning of the metal takes place and neglecting the corner radius, calculate the depth of flange made from the 4 in. allowance.
21. An oil tank is cylindrical with semi-spherical ends. It is 12 in. in diameter and the overall length is 6 ft. Calculate (*a*) the volume of the tank, (*b*) how many gallons of oil it will hold if 1 cu. ft holds 6·25 gallons.
22. A water tank is elliptical in cross-section. Its major axis is 24 in., its minor axis is 18 in. and its total length is 7 ft. Calculate (*a*) the volume of the tank, (*b*) how many gallons of water it will hold if 1 cu. ft holds 6·25 gallons.

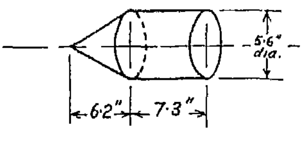


FIG. 57

23. Calculate the volume of the funnel shown in Fig. 56.
24. Calculate the volume of the container shown in Fig. 57.

TRIGONOMETRY

By use of the Theorem of Pythagoras a right angled triangle can be solved with respect to the lengths of its sides. However, many practical problems involve the angles of a triangle as well as its sides, and this introduces the use of trigonometry.

Consider any right angled triangle ABC (Fig. 58).

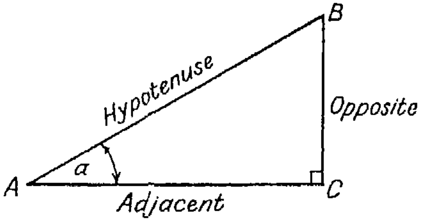


FIG. 58

The angle at C is the right angle and AB is the hypoteneuse. Side AC is beside or adjacent to the angle a and side BC is opposite the angle a. In this way the sides of a triangle are named with reference to one of the acute angles.

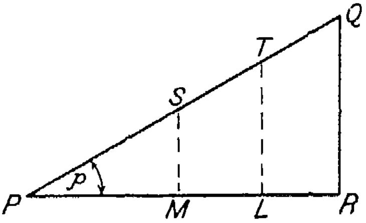


FIG. 59

In the right angled triangle PQR (Fig. 59), the ratio of the length of sides QR and PQ is $\frac{QR}{PQ}$. With reference to the angle p this ratio is, $\frac{\text{Opposite}}{\text{Hypoteneuse}} = \frac{QR}{PQ}$.

If a perpendicular TL is drawn, then in the triangle PTL the ratio is $\frac{\text{Opposite}}{\text{Hypoteneuse}} = \frac{TL}{PT}$. The angle ($p$) is still the same but the opposite and hypoteneuse sides have been decreased in proportion to each other. However the ratio $\frac{\text{Hypoteneuse}}{\text{Opposite}}$ has the same value for both triangles

$$\frac{QR}{PQ} = \frac{TL}{PT}$$

This is true also for the triangle PSM and sides $\frac{\text{SM}}{\text{PS}}$.

Thus for any right angled triangle the ratio of sides $\frac{\text{Opposite}}{\text{Hypoteneuse}}$ will be constant and in trigonometry this ratio is called the sine ratio:

$$\therefore \text{ sine of angle } a = \frac{\text{Opposite}}{\text{Hypoteneuse}}.$$

In the same way the ratios of the other sides will produce a constant and these are called:

$$\frac{\text{Adjacent}}{\text{Hypoteneuse}} \text{ ratio} = \text{cosine of angle } a.$$

$$\frac{\text{Opposite}}{\text{Adjacent}} \quad \text{ratio} = \text{tangent of angle } a.$$

The numerical values of these ratios depends only on the size of the angle concerned.

Summary

There are three primary ratios in trigonometry:

$$\text{sine } a \quad \text{or } \sin a = \frac{\text{Opposite}}{\text{Hypoteneuse}}$$

$$\text{cosine } a \quad \text{or } \cos a = \frac{\text{Adjacent}}{\text{Hypoteneuse}}$$

$$\text{tangent } a \text{ or } \tan a = \frac{\text{Opposite}}{\text{Adjacent}}.$$

The numerical values of these ratios have been calculated for all angles and can be found in trigonometry tables under the headings, ‘ natural sines ’, ‘ natural cosines ’, ‘ natural tangents ’ or sometimes ‘ functions of angles ’.

There is, of course, another acute angle in the triangle and it also has its own adjacent and opposite sides. Fig. 60 shows the names of the sides with respect to each angle.

For each angle the respective ratios are constant.

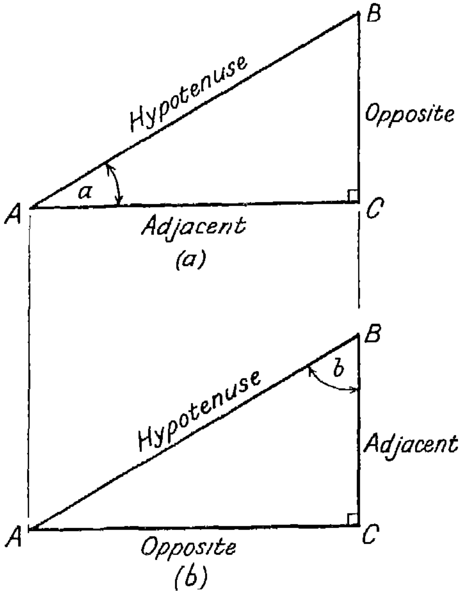


FIG. 60

Trigonometry Tables

The tables are used in the same way as logarithm tables with the exception that in the cosine table the mean differences are subtracted instead of added.

EXAMPLE. *From the tables find the values (a) sine* 57° 27′, *(b) cosine* 28° 52′, *(c) tangent* 74° 23′.

(*a*) Sine 57° 27′

sin 57° 24′ = 0·8425
add mean difference 3′ = 0·0005
∴ sin 57° 27′ = 0·8430

(*b*) Cosine 28° 52′

cos 28° 48′ = 0·8763
subtract mean difference 4′ = 0·0006
∴ cos 28° 52′ = 0·8757

(*c*) Tangent 74° 23′

tan 74° 18′ = 3·5576
add mean difference 5′ = 0·0204
∴ tan 74° 23′ = 3·5780

Notes

The sine ratios range from 0·0000 for 0° to 1·0000 for 90°
cosine ratios range from 1·0000 for 0° to 0·0000 for 90°
tangent ratios range from 0·0000 for 0° to 573 for 89° 54′.

Be sure to carry any whole number given in the first column of the tangent ratios into the body of the tables giving the minute part of the angle.

EXAMPLE 1. *Calculate the values of the angles* a *and* b *in the triangle shown in Fig. 61 if AB is 8 in. long and BC is 3·5 in. long and the angle at C is a right angle.*

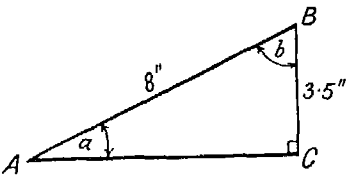


FIG. 61

Angle *a*

The sides involved are the opposite and the hypoteneuse therefore the sine ratio is used.

$$\sin a = \frac{\text{opposite}}{\text{hypoteneuse}} = \frac{3{\cdot}5}{8} = 0{\cdot}4375$$

from the sine tables, angle $a = \underline{25^\circ\ 57'}$.

Since there are 180° in a triangle and angle at C is 90°

$$\begin{aligned} \text{then angles } a + b &= 90^\circ \\ \text{and angle } b &= 90^\circ - a \\ &= 90^\circ - 25^\circ\ 57' = \underline{64^\circ\ 3'} \end{aligned}$$

or: for angle *b* the known sides are the adjacent and the hypoteneuse

$$\begin{aligned} \text{using the cosine ratio: } \cos b &= \frac{\text{adjacent}}{\text{hypoteneuse}} = \frac{3{\cdot}5}{8} \\ &= 0{\cdot}4375 \end{aligned}$$

from the cosine tables, angle $b = 64^\circ\ 3'$ as before.

EXAMPLE 2. *In the triangle shown in Fig. 62 the angle* a *is 27° 35′, the side AC is 9 in. long and the angle at C is a right angle. Calculate the length of side BC.*

The sides to be used with the angle *a* are the opposite and the adjacent, therefore the tangent ratio is used.

$$\begin{aligned} \tan a &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{\text{BC}}{\text{AC}} \\ \therefore\ \tan a &= \frac{\text{BC}}{9} \\ \text{and BC} &= 9 \times \tan a \\ &= 9 \times 0{\cdot}5224 = 4{\cdot}7016 \text{ in.} \\ &\underline{\text{say } 4{\cdot}7 \text{ in.}} \end{aligned}$$

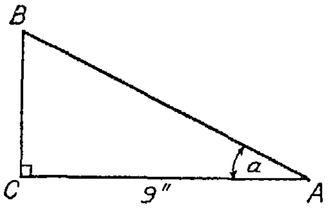


FIG. 62

EXAMPLE 3. *From the triangle shown in Fig. 63 calculate the length of side QR if the angle r is 49° and the angle at P is a right angle and PR is 6·5 in.*

With angle *r* the sides to be used are the adjacent and the hypoteneuse. The cosine ratio is used.

$$\cos 49^\circ = \frac{6{\cdot}5}{QR}$$

$$\therefore \quad QR = \frac{6{\cdot}5}{\cos 49^\circ}$$

$$= \frac{6{\cdot}5}{0{\cdot}6561} = 9{\cdot}906 \text{ in.}$$

$$\underline{\text{say } 9{\cdot}9 \text{ ins.}}$$

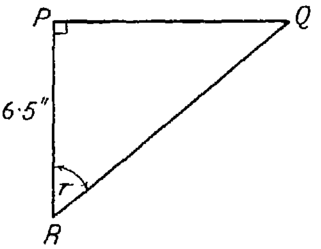


FIG. 63

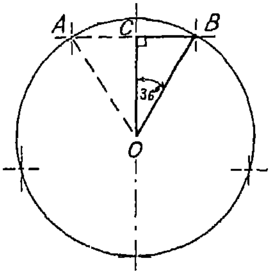


FIG. 64

EXAMPLE 4. *Five holes, equally spaced apart, have to be drilled in a plate around a 9-in. diameter circle. Calculate the distance between the centres of two adjacent holes (Fig. 64).*

In a circle there are 360°.

In one fifth of a circle $\frac{360}{5}^\circ = 72^\circ$.

∴ In the triangle ABO the angle at O = 72°.

In ABO there is no right angle and the trigonometry ratios cannot be used.

From C, the mid-point of AB, drop a perpendicular to meet O.

This will bisect the angle at O, and make two right angles at C.

In the triangle CBO,

$$\text{The angle at O is } \frac{72}{2}^\circ = 36^\circ.$$

$$\text{OB is a radius} = 4{\cdot}5 \text{ in.}$$

$$\sin 36^\circ = \frac{CB}{4{\cdot}5}$$

$$\therefore \quad CB = 4{\cdot}5 \times \sin 36^\circ$$

$$= 4{\cdot}5 \times 0{\cdot}5878 = 2{\cdot}645 \text{ in.}$$

$$\therefore \quad \text{centre distance AB} = 2 \times 2{\cdot}645 \text{ in.}$$

$$= \underline{5{\cdot}29 \text{ in.}}$$

EXAMPLE 5. *A regular octagon has to be cut from a round disc 4 in. diameter. Calculate the length of the flats (Fig. 65).*

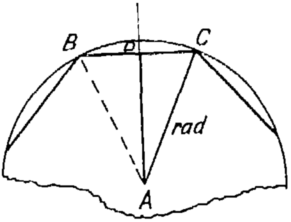


FIG. 65

An octagon has 8 sides, and a circle contains 360°.

$\therefore$ angle at the centre of each side $= \frac{360°}{8} = 45°$

and the angle at A in the triangle ADC $= \frac{45}{2} = 22\frac{1}{2}°$

$$\sin 22\tfrac{1}{2}° = \frac{DC}{AC}$$

$$\therefore \quad DC = AC \sin 22\tfrac{1}{2}°$$

$$= 2 \times \cdot 3827 = \cdot 7654 \text{ in.}$$

and length of flat BC $= 2 \times \cdot 7654$ in.

$= \underline{1 \cdot 53 \text{ in.}}$

EXAMPLE 6. *A tank is to be made to the cross-section shown in Fig. 66. Calculate the length of metal required for the perimeter neglecting laps.*

Fig. 67 shows the working diagram.

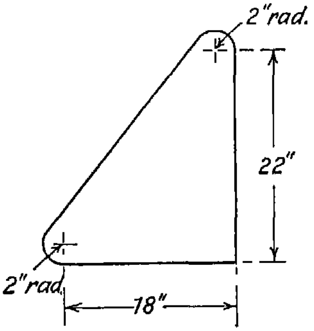


FIG. 66

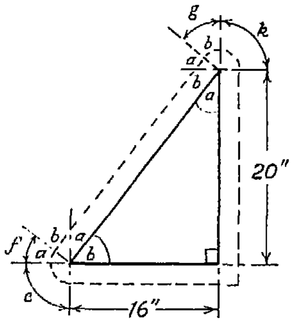


FIG. 67

$\tan b = \frac{20}{16} = \frac{5}{4} = 1 \cdot 25$

$\therefore \quad b = 51° \, 20'$

and $a = 90° - 51° \, 20' = 38° \, 40'$.

To find the sloping side length,

$$\sin 51^\circ\ 20' = \frac{20}{\text{Hypoteneuse}}$$

$$\therefore\ \text{hypoteneuse} = \frac{20}{\sin 51^\circ\ 20'}$$

$$= \frac{20}{\cdot 7808} = 25{\cdot}62 \text{ in.}$$

Now angles a and b are complementary, that is they add up to a right angle 90°.

By going around the angles at the radii corners it is seen that at the top there is a quarter circumference plus the arc of angle $b : k + g$. At the bottom a quarter circle plus angle $a : e + f$.

$$\therefore\ \text{The curved top part has} = 90^\circ + 51^\circ\ 20' = 141^\circ\ 20'$$

$$\text{and the bottom} = 90^\circ + 38^\circ\ 40' = 128^\circ\ 40'$$

a circumference encloses $360^\circ = \pi \times$ dia.

the curved length encloses $128^\circ\ 40' + 141^\circ\ 20' = 270^\circ$

$$\text{the arc for } 270^\circ = \pi d \times \tfrac{270}{360}$$

$$= \pi \times 4 \times \tfrac{270}{360}$$

$$= 3\pi = 9{\cdot}42 \text{ in.}$$

$$\therefore\ \text{total perimeter} = 18 + 22 + 25{\cdot}62 + 9{\cdot}42$$

$$= \underline{75{\cdot}04 \text{ in.}}$$

EXAMPLE 7. *Fig.* 68 *shows the elevation of a system of ductwork. Calculate the angle of offset* (α) *and the length of centre line AB.*

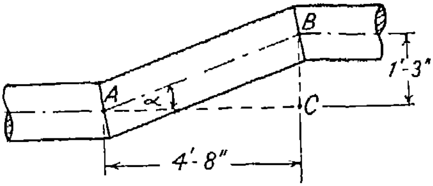


FIG. 68

$$\frac{BC}{AC} = \tan \alpha$$

$$\frac{15}{56} = \tan \alpha$$

$$\text{From which } \tan \alpha = 0{\cdot}2678$$

$$\text{and angle of offset } \alpha = \underline{15^\circ}$$

$$\frac{AC}{AB} = \cos 15^\circ$$

$$\text{Then AB} = \frac{56}{0{\cdot}9659} \text{ by transposition}$$

$$= \underline{57{\cdot}98 \text{ in.}} \text{ or } \underline{4 \text{ ft } 10 \text{ in.}} \text{ (approx.).}$$

Example 8. *A right cone has an included angle of* 57° *at the apex and a base diameter of* 1 *ft* 8 *in. Calculate the weight of the cone if it is made from material weighing* 5 *lb/sq. ft* (Fig. 69).

$$\frac{10}{\text{Slant height}} = \sin 28^\circ\, 30'$$

$$\therefore \quad \text{Slant height} = \frac{10}{\sin 28^\circ\, 30'} = \frac{10}{0{\cdot}4772} = 20{\cdot}95 \text{ in.}$$

By formula,

$$\begin{aligned}\text{Curved surface area} &= \text{Mid circumference} \times \text{Slant ht.}\\ &= \pi \times 10 \times 20{\cdot}95\\ &= 657{\cdot}83 \text{ sq. in.}\\ \text{Weight} &= \frac{657{\cdot}83 \times 5}{144}\\ &= \underline{22{\cdot}84 \text{ lb.}}\end{aligned}$$

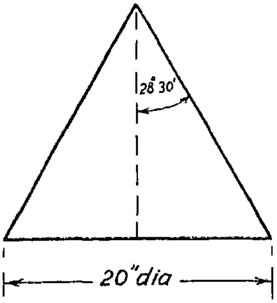


Fig. 69

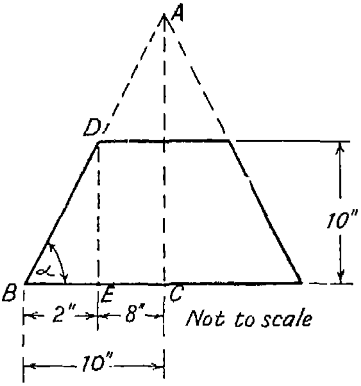


Fig. 70

Example 9. *A frustum of a right cone is* 20 *in. dia. at the base,* 16 *in. dia. at the top and* 10 *in. vertical height. Calculate the major and minor radii for the pattern* (*Fig.* 70).

$$\text{In triangle DBE} \quad \frac{\text{DE}}{\text{BE}} = \tan \alpha$$

$$\text{then} \quad \frac{10}{2} = 5 = \tan \alpha$$

$$\text{from which } \alpha = 78^\circ\, 42'$$

$$\text{Also, in triangle ABC} \quad \frac{\text{BC}}{\text{AB}} = \cos \alpha$$

$$\text{then} \quad \frac{10}{\text{AB}} = \cos \alpha = 0{\cdot}1959$$

$$\text{from which AB} = \frac{10}{0{\cdot}1959} = 51{\cdot}04 \text{ in.}$$

$$\therefore \quad \text{Major radius} = \underline{51{\cdot}04 \text{ in.}}$$

$$\text{Similarly } \frac{BE}{DB} = \cos \alpha$$

$$\text{then } \frac{2}{DB} = \cos \alpha = 0{\cdot}1959$$

$$\text{from which } DB = \frac{2}{0{\cdot}1959}$$

$$\therefore \quad \text{Minor radius} = \underline{10{\cdot}2 \text{ in.}}$$

EXAMPLE 10. *20 blanks of the shape shown in Fig. 71 are to be cut from a sheet 6 ft × 4 ft. Calculate the percentage of metal remaining.*

It is first necessary to calculate the vertical height h.

$$\frac{h}{AB} = \sin 65^\circ$$

$$\text{from which } h = 7 \times {\cdot}9063$$

$$= 6{\cdot}3441 \text{ in.}$$

$$\text{Then, area of triangle} = \frac{\text{Base} \times \text{ht.}}{2}$$

$$= \frac{10 \times 6{\cdot}3441}{2} = 31{\cdot}7205 \text{ sq. in.}$$

$$\text{Area of 20 blanks} = 31{\cdot}7205 \times 20$$

$$= 634{\cdot}41 \text{ sq. in.}$$

$$\text{Area of sheet} = 6 \times 4 \times 144$$

$$= 3456 \text{ sq. in.}$$

$$\text{Percentage remaining} = \frac{\text{Area remaining}}{\text{Total area}} \times \frac{100}{1}$$

$$= \frac{3456 - 634{\cdot}4}{3456} \times \frac{100}{1} = \frac{2821{\cdot}6}{3456} \times \frac{100}{1}$$

$$= \underline{81{\cdot}64} \text{ per cent.}$$

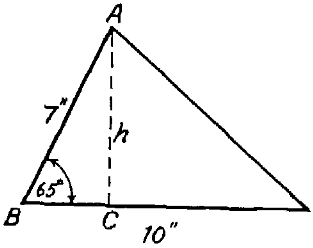


FIG. 71

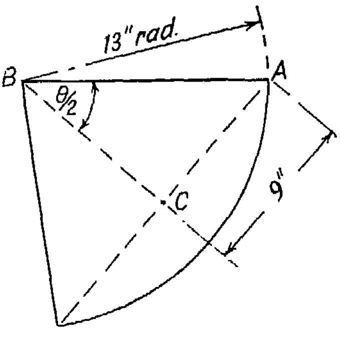


FIG. 72

EXAMPLE 11. *The pattern for a right cone is in the form of a sector of a circle 13 in. radius, and the chord across the arc measures* 18 *in.* (Fig. 72).

Calculate (a) the included angle θ for the pattern, (b) the volume of the cone.

Bisecting the angle θ will form the right angle triangle ABC.

$$\text{Hence } \frac{\text{AC}}{\text{AB}} = \sin\frac{\theta}{2}$$

$$\therefore \quad \tfrac{9}{13} = \sin\frac{\theta}{2}$$

$$\text{and } \sin\frac{\theta}{2} = 0{\cdot}6923$$

$$\text{Therefore angle } \frac{\theta}{2} = 43^\circ\,49' \text{ (from tables)}$$

$$\text{and angle } \theta = \underline{87^\circ\,38'}$$

$$\text{By formula, } \theta = \frac{180\text{D}}{\text{R}} \text{ (see page 71)}$$

$$87{\cdot}63^\circ = \frac{180^\circ \times \text{D in.}}{13 \text{ in.}}$$

$$\text{from which diameter (D)} = \frac{87{\cdot}63^\circ \times 13 \text{ in.}}{180^\circ}$$

$$= 6{\cdot}327 \text{ in.}$$

$$\text{vertical height of cone } (h) = \sqrt{[(\text{Pattern rad.})^2 - (\text{base rad.})^2]}$$

$$= \sqrt{(13^2 - 3.1625^2)}$$

$$= \sqrt{(169 - 10)} = \sqrt{159} = 12{\cdot}61 \text{ in.}$$

$$\text{Volume of cone} = \frac{\pi r^2 h}{3}$$

$$= \frac{\pi \times 3{\cdot}1625^2 \times 12{\cdot}61}{3} = \underline{132} \text{ cu. in.}$$

Exercise 13

1. From the tables find the value of: (i) tan 41° 33′, (ii) tan 71° 50′, (iii) tan 13° 16′.
2. Find the value of: (i) sin 24° 39′, (ii) sin 43° 9′, (iii) sin 76° 20′.
3. Find the value of: (i) cos 63° 34′, (ii) cos 12° 45′, (iii) cos 73° 34′.
4. Find the angle whose tan is: (i) 2·2148, (ii) 1·1197, (iii) 2.6770.
5. Find the angle whose sin is: (i) 0·1100, (ii) 0·9047, (iii) 0·3018.
6. Find the angle whose cos is: (i) 0·6622, 0·2056, (iii) 0·9644.

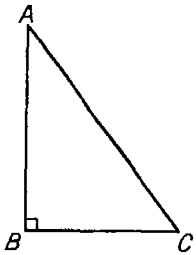


FIG. 73

7. The triangle ABC in Fig. 73 is right angled at B.
   (i) If AB is 11·5 in. and the angle at C is 49° find the length of BC.
   (ii) If BC is 8·39 in. and the angle at C is 50° find the length of AB.
   (iii) If AB is 5·8 in. and BC is 2 in. find the angle at C.

8. The triangle RST in Fig. 74 is right angled at S.

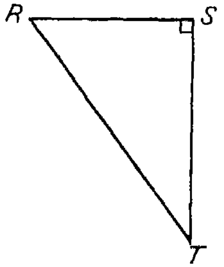


FIG. 74

(i) If RT is 3·5 in. and the angle at T is 27° find the length of RS.

(ii) If RT is 8 in. and the angle at T is 36° find the length of ST.

(iii) If RS is 4 in. and RT is 7 in. find the angle at T.

9. An octagon is cut from a circle of $1\frac{7}{8}$ in. dia. Calculate the distance between the flats.
10. A 9-in. diameter flue pipe has to pass through a roof which is inclined at 35° to the horizontal. The hole in the roof is an ellipse, calculate the lengths of the major and minor axes.
11. A tank has to be made to the cross-section shown in Fig. 75. Calculate the length of metal required for the perimeter, neglecting laps.
12. A roof apex cover in the form of a square pyramid has an included angle of 64° between the flat faces of opposite sides. If the base measures 26 in. along each side, calculate the weight of the cover if made from metal weighing 10 lb per square ft.
13. A frustum of a right cone is 24 in. diameter at the top, 18 in. diameter at the base and 15 in. vertical height. Calculate the major and minor radii for the pattern.

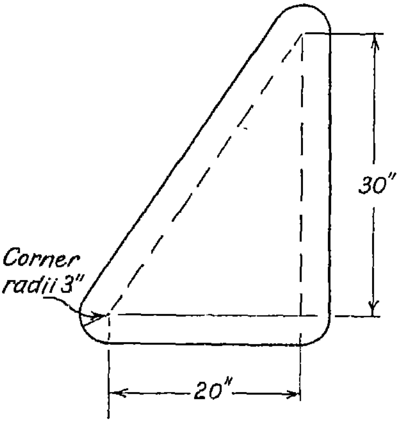


FIG. 75

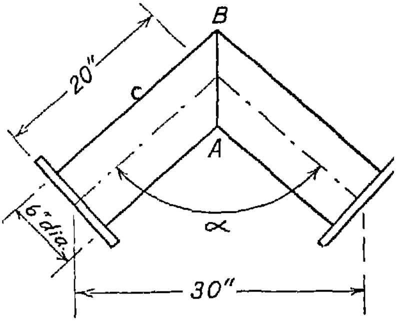


FIG. 76

14. A mild steel plate $\frac{1}{4}$ in. thick is in the form of a triangle ABC. The side AB measures 21 in. and BC measures 42 in.; the angle at B is 50°. Calculate the weight of the plate if made from mild steel weighing 0·28 lb per cubic inch.
15. The pattern for a right cone in the form of a sector of a circle has an included angle of 120° and the chord across the arc measures 26 in. Calculate (*a*) the pattern radius, (*b*) the vertical height of the cone.
16. Calculate the angle of offset α and the length of welding required around the joint line AB for the elbow shown in Fig. 76.

Force

In the workshop, metals are worked by hammering, bending, cutting, etc. In all these operations *force* is applied on the metal. If we study the action and effects of these forces we shall be able to understand the natural laws which govern them and use our tools more effectively.

A force, we know from experience, usually causes movement of the part to which it is applied, but often the force is not large enough to cause motion (movement) although it *tends* to do so.

Force can therefore be defined as: **Force is whatever causes, or tends to cause, motion of a body.**

The unit of force is the **pound** or **ton** and is the amount of force which would be used if a pound or ton weight were applied to a part.

Types of Force

No matter what the origin of the force, we are interested in its *effect* on the part to which it is applied:

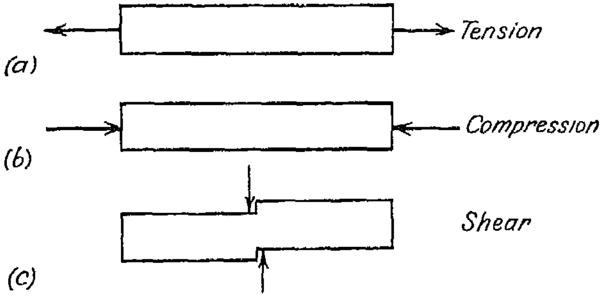


FIG. 77

(1) If the force tends to pull the object apart, it is called a *tensile* force and the part is said to be in *tension* (Fig. 77(a)).
(2) If it tends to squeeze the part, it is a *compressive* force and the part is in *compression* (Fig. 77(b)).
(3) If it tends to displace one part of the metal sideways from the rest, it is a *shearing* force and the part is in *shear* (Fig. 77(c)).

When a force is applied to a metal part, the part will resist the force up to a certain limit, and when the force is removed the metal will spring back to its former shape and size, showing that the metal is *elastic*. If the force is now increased beyond the limit, the metal will no longer be able to remain elastic and resist the force; instead it will take on a *permanent set* and remain stretched or bent. If the force is increased still more the metal will fail and fracture.

The force applied, whether or not it causes a *permanent set*, will *stress* the metal. The intensity of stress will depend upon:

(*a*) the amount of force applied,

(*b*) the area of the cross-section resisting the force.

Stress is therefore expressed as,

$$\text{Stress} = \frac{\text{Force applied, lb or ton}}{\text{Cross-section area, in.}^2}$$

The units of stress are lb/in.^{2} or ton/in.^{2}

EXAMPLE 1. *A sheet metal flue box weighs 250 lb and is supported by two stay rods of ½ in. dia. Calculate the stress in each rod.*

$$\text{Cross-section area of rod} = \pi r^2 = \pi \tfrac{1}{16} = 0{\cdot}196 \text{ in.}^2$$

Assume each rod carries half the load;

$$\begin{aligned} \text{each rod carries } \tfrac{250}{2} &= 125 \text{ lb} \\ \text{Stress in each rod} &= \frac{\text{load, lb}}{\text{area, in.}^2} \\ &= \frac{125}{0{\cdot}196} = \underline{637{\cdot}7 \text{ lb/in.}^2} \end{aligned}$$

EXAMPLE 2. *Calculate the force required to punch out* 2-*in. dia. blanks from strip* 0·252 *in. thick* (3 S.W.G.) *if the ultimate shear stress is* 18 *tons/in.*^{2}

(The *ultimate stress* of a metal is the greatest stress the metal can withstand and is therefore in effect the stress required to shear the metal.)

The area sheared is the circumference of the blank × its thickness.

$$\begin{aligned} \text{Area sheared} &= \pi d \times \text{thickness} \\ &= 2\pi \times {\cdot}252 = 1{\cdot}58 \text{ in.}^2 \\ \text{Stress} &= \frac{\text{Force}}{\text{Area}} \text{ tons/in.}^2 \\ \therefore \quad \text{Force} &= \text{stress} \times \text{area} \\ &= 18 \times 1{\cdot}58 = \underline{28{\cdot}44 \text{ tons.}} \end{aligned}$$

EXAMPLE 3. *A single riveted lap joint has* 10 *equally spaced rivets each* ¼ *in. dia. If the maximum shearing stress is limited to* 7000 *lb/in.*^{2}, *calculate the maximum safe load for the joint.*

$$\begin{aligned} \text{Stress} &= \frac{\text{load}}{\text{area}} \text{ lb/in.}^2 \\ \therefore \quad \text{load} &= \text{stress} \times \text{area} \\ \text{total area} &= 10 \times \pi r^2 = 10 \times 3{\cdot}14 \times ({\cdot}125)^2 \\ &= 10 \times 0{\cdot}049 = 0{\cdot}49 \text{ in.}^2 \\ \text{and load} &= 7000 \times 0{\cdot}49 \\ &= \underline{3430 \text{ lb.}} \end{aligned}$$

EXAMPLE 4. *A hollowing hammer falls on a piece of copper. The force of the blow is estimated to be 40 lb and the contact diameter of the hammer face is 0·75 in. Calculate the compressive stress induced in the metal.*

$$\text{Stress} = \frac{\text{load}}{\text{area}}\ \text{lb/in.}^2$$

$$= \frac{40}{3{\cdot}14 \times ({\cdot}375)^2} = \underline{90{\cdot}9\ \text{lb/in.}^2}$$

Moment of a Force

Another important effect of a force acting on a body is to cause, or tend to cause, the body to rotate. This turning effect is called the *moment of a force*. When a force is applied to the end of a spanner to turn a nut, to press on the handles of snips to cut tinplate, or to turn a screw with a screwdriver, the moment of the force is being used. In each case the force causes rotation about a point, called the *fulcrum*.

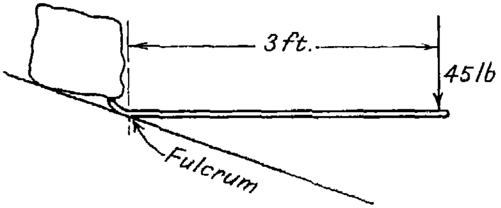


FIG. 78

The magnitude of the moment is measured by the product of the force and the **perpendicular** distance from the fulcrum to the line of action of the force.

Moment = force, lb × distance, in. or ft.

The units are lb in. or lb ft.

A simple application of moments is shown in the use of a crowbar to raise a large load with a small effort. The effort is moved through a large distance and the load is raised a small distance (Fig. 78).

$$\text{Moment at the handle} = \text{force, lb} \times \text{perpendicular distance, ft}$$

$$= 45 \times 3 = 135\ \text{lb ft.}$$

If the distance from the fulcrum to the point of the crowbar is 2 in. = $\frac{1}{6}$ ft; and with the same moment effective at the point to lift the load, then

$$\text{Moment} = 135 \text{ lb ft} = \text{force} \times \text{distance}$$
$$= \text{force} \times \tfrac{1}{6}$$
$$\text{and force} = \frac{135}{\frac{1}{6}} = 135 \times 6$$
$$= 810 \text{ lb.}$$

Thus using a crowbar with a ratio of 3 ft to $\frac{1}{6}$ ft an effort of 45 lb can raise a load of 810 lb.

EXAMPLE 5. *A man applies a force of* 48 *lb to the wheel of a spinning jenny* 9 *in. dia. Calculate the effective turning moment.*

$$\text{Force} = 48 \text{ lb}$$
$$\text{Distance} = \text{radius of wheel} = 4\tfrac{1}{2} \text{ in.} = \frac{4\frac{1}{2}}{12} \text{ ft}$$
$$\text{Moment} = \text{Force, lb} \times \text{distance, ft}$$
$$= 48 \times \frac{4\frac{1}{2}}{12} = 48 \times \frac{9}{2 \times 12} \text{ lb ft}$$
$$= \underline{18 \text{ lb ft.}}$$

This moment may have been expressed as:

$$\text{Moment} = 48 \text{ lb} \times 4\tfrac{1}{2} \text{ in.}$$
$$= \underline{216 \text{ lb in.}}$$

EXAMPLE 6. *The force required to turn the handle of a* 10-*in. spanner is* 70 *lb. Calculate the additional force required with a* 7½-*in. spanner.*

Turning moment required = 70 lb × 10 in. = 700 lb in.

Assuming the same moment is required with the 7½-in. spanner:

$$\text{Moment} = \text{force} \times \text{distance}$$
$$700 = \text{force} \times 7\tfrac{1}{2} \text{ in.}$$
$$\text{and force} = \frac{700}{7\frac{1}{2}} = \frac{700 \times 2}{15} \text{ lb}$$
$$= 93\tfrac{1}{3} \text{ lb}$$
$$\therefore \text{ additional force} = 93\tfrac{1}{3} - 70 = \underline{23\tfrac{1}{3} \text{ lb.}}$$

EXAMPLE 7. *Using a pair of metal shears, the force required to cut the plate is* 126 *lb at a distance of* 1 *in. from the fulcrum rivet. Calculate the force required at the handle* 5 *in. from the rivet* (Fig. 79).

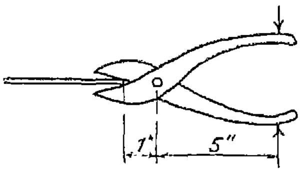


FIG. 79

Cutting moment at blade = 126 × 1 = 126 lb/in. The same moment is required at the handle.

$$\text{Moment} = \text{force} \times \text{distance}$$
$$126 = \text{force} \times 5 \text{ in.}$$
$$\text{Force} = \frac{126}{5} = 25{\cdot}2 \text{ lb.}$$

The Principle of Moments

Since a moment may cause a part to turn to the left (anticlockwise) or to the right (clockwise) it is classified as clockwise or anticlockwise. The important Principle of Moments deals with forces acting on a part which remains in equilibrium, i.e., it does not move or moves at a constant speed. The principle states that **if a body is acted upon by a number of forces and it remains in equilibrium, then the sum of the clockwise moments equals the sum of the anticlockwise moments.**

This principle is applied to solve many problems. The method is to *take moments* about a point on the part. This means that the point, about which moments are to be taken, is considered to act like a hinge, and every force outside this point will tend to make the part turn about this point. It is important to realise that if a force acts *at* the point about which moments are taken this force has *no moment*; which fact is obvious since, although there is a force, as it acts on the point there is no perpendicular distance. The moment then is: force × zero distance = 0.

EXAMPLE 8. *A girder* 16 *ft long weighs* 400 *lb and supports* 3 *sheet metal working machines A, B and C. The girder itself rests on vertical end supports. If the respective weights of the machines are* 150 *lb,* 200 *lb and* 350 *lb, and they are placed at distances of* 4 *ft,* 9 *ft and* 12 *ft respectively from the left-hand support, determine the load carried by each end support. (Assume that the girder weight acts at the mid point G)* (Fig. 80).

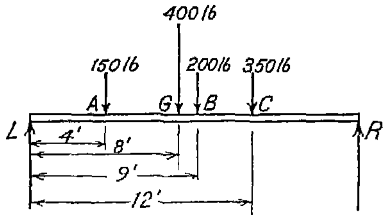


FIG. 80

The end supports must have a reaction upwards to counteract the total downward load.

Taking moments about the left-hand end (L),

Clockwise moments = (150 × 4) + (400 × 8) + (200 × 9) + (350 × 12) lb/ft
= 600 + 3200 + 1800 + 4200
= 9800 lb/ft.

Anticlockwise moments = reaction at R × 16 ft
= 16R

now clockwise = anticlockwise moments
∴ 9800 = 16R

$R = \frac{9800}{16} = \underline{612{\cdot}5 \text{ lb.}}$

If the R.H. support carries 612·5 lb of the load, the L.H. support must carry:

Total load − 612·5 lb
1100 − 612·5
= 487·5 lb.

The answer could be checked by taking moments about the R.H. support from which the load at the L.H. support could be calculated.

Taking moments about R

$$L \times 16 = (350 \times 4) + (200 \times 7) + (400 \times 8) + (150 \times 12)$$

$$16L = 7800$$

$$L = \frac{7800}{16} = \underline{487{\cdot}5 \text{ lb}} \text{ as before.}$$

Machines

A machine is a device by which a small force applied at one end can produce a large force at the other. Snips, pulley blocks, hand shears and of course the lever or crowbar are all examples of simple machines.

The crowbar shows the principle of all machines; the small force moving through a large distance enables a large load to be moved through a short distance.

The ratio of these distances moved can be called the **distance ratio** or, since one is generally concerned with speeds when dealing with machines, more usually the **velocity ratio.**

$$\text{Velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

Normally the ratio is greater than 1, but if it is agreed to have an output force less than the force put in, so as to obtain more speed, the ratio is less than 1, as in the case of the bicycle.

When a machine is used an advantage is gained over a load, called *mechanical advantage.* Mechanical advantage is the ratio of load to effort:

$$\text{Mechanical advantage} = \frac{\text{Load}}{\text{Effort}}$$

Even in the simplest machine, a pair of snips for example, some of the work put into it is lost in overcoming friction at the hinge pin. No machine could be 100 per cent. efficient since there is always some friction to be overcome.

$$\text{Efficiency of a machine} = \frac{\text{Output}}{\text{Input}}$$

Output and input refer to the work done and

$$\text{Work done (ft lb)} = \text{Force lb} \times \text{distance moved by force ft.}$$

EXAMPLE 9. *A screw press has a handle of* 12 *in. effective diameter. For one turn of the handle the screw advances* $\frac{3}{4}$ *in. If the press delivers a force of* 760 *lb with an applied effort of* 80 *lb, calculate the velocity ratio mechanical advantage and efficiency of the press.*

Let the effort move the handle through 1 revolution.

It will move a circumference $= \pi D = \pi 12$ in.

The load will move ¾ in.

$$\begin{aligned}
\text{Velocity ratio} &= \frac{\text{distance moved by effort}}{\text{distance moved by load}} = \frac{12\pi}{\frac{3}{4}} = \frac{4 \times \pi 12}{3} = \underline{50.24} \\
\text{Mechanical advantage} &= \frac{\text{load}}{\text{effort}} = \frac{760}{80} = \underline{9{\cdot}5} \\
\text{Efficiency} &= \frac{\text{output}}{\text{input}} = \frac{\text{load} \times \text{distance moved by load}}{\text{effort} \times \text{distance moved by effort}} \\
&= \frac{760 \times \frac{3}{4}}{80 \times \pi 12} \\
&= 0{\cdot}189 \\
\text{Efficiency per cent} &= 0{\cdot}189 \times 100 = \underline{18{\cdot}9 \text{ per cent.}}
\end{aligned}$$

Exercise 14

1. A blanking punch is of rectangular cross-section, 2 in. × 1¾ in. While in action it is subjected to a resisting force in the metal of 10·5 tons. Calculate the stress in the punch and state whether it is in tension or compression.
2. A ½-in. dia. mild steel rod is being cut up on a shearing machine. If the ultimate shear strength or stress of the material is 22 ton/in.^{2}, calculate the force on the blade.
3. Three ¾-in. dia. tie rods support an extractor pipe. If the pipe weighs 240 lb, calculate the stress in each rod assuming that the load is equally shared.

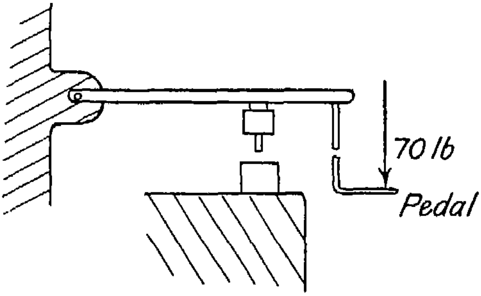


FIG. 81

4. 1¾-in. dia. discs are to be blanked out of ⅛-in. thick mild steel sheet. If the shear strength of the sheet is 20 tons/in.^{2}, calculate the shearing force required.
5. Explain the mechanical principle of cutting sheet metal by the use of snips. If the force required to shear a metal is x lb, what force will be required if the length of the arms of the snips measured from the rivet is doubled?
6. A small punch is foot operated. The distance from the fulcrum to the pedal rod is 14 in. and from the fulcrum to the punch 8 in. If the lever is depressed with a force of 70 lb calculate the force at the punch (Fig. 81).
7. The lever shown operates a return mechanism of a machine. If the spring exerts a pull of 35 lb calculate the force in the rod (Fig. 82).

8. A screw press has a handle of 10 in. dia. The screw advances $\frac{1}{2}$ in. for each turn of the handle. Calculate the velocity ratio of the press. If the mechanical advantage is 10 what effort will be required to deliver a force of 105 lb?

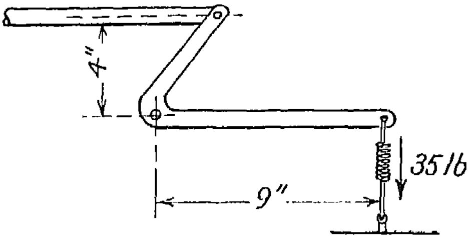


FIG. 82

9. A machine can raise a load of 375 lb through 3 inches with an effort of 18 lb which moves through 8 ft. Calculate the velocity ratio, mechanical advantage and efficiency of this machine.
10. A lifting block has a velocity ratio of 35 to 1, and a mechanical advantage of 14. Calculate the distance the effort must move through to raise a load of 86 lb through 22 inches. How much effort will be required?

Properties of Metals

(*a*) **Tenacity or Tensile Strength** is resistance to fracture by a stretching force. It is possessed by all metals to a greater or lesser degree, and is expressed by the maximum dead weight load that a part of known cross-section area can support without rupture. In Britain this resistance is expressed as a number of pounds or tons that a bar, one square inch in area, can withstand, e.g., pounds or tons/in^2.

Tenacity may therefore be defined as the property a metal possesses to resist its particles being torn apart.

This property is greatly affected by the purity, structure and temperature of the metal and the treatment it has received. Certain impurities tend to increase the tensile strength of a metal, while others diminish it. Generally the tenacity of a metal is considerably reduced by marked increase in temperature. For example, mild steel at normal temperature has a tensile strength of 28 tons per sq. in. At 600°C this is reduced to 10 tons per sq. in., and at 900°C it is only 6 tons per sq. in.

(*b*) **Malleability** is the property of a metal to be beaten or rolled in all directions. The degree of malleability may be measured by the thinness of the leaves or foil it is possible to produce by rolling without rupture or cracking. This property of a metal may be drastically altered by the inclusion of certain impurities, and in some instances a trace of a certain substance is sufficient to destroy malleability. Even gold is made brittle by traces of bismuth, arsenic and antimony.

(*c*) **Ductility** is the property of a metal to be drawn in the direction of its length, that is, converted into wire. The metal from which the finest wire is produced is said to be the most ductile.

A metal must be tenacious and malleable to be ductile, and any condition which tends to reduce either of these properties will also lower the ductility. For this reason all wire is drawn when the metal is cold since heating reduces the tenacity.

Metal	*Tensile strength Tons per sq. in.*
Mild steel	25–30
Copper	12–15
Aluminium	7–10
Zinc	3
Tin	2·1
Lead	1·5

Descending order of ductility: mild steel, copper, aluminium, zinc, tin, lead.

Descending order of malleability: lead, tin, aluminium, copper, mild steel, zinc.

Density. The density of a material is the weight of a unit volume. The units are lb per cubic inch.

$$\text{Density} = \frac{\text{Weight of part (lb)}}{\text{Volume of part (in.}^3)}, \ \text{lb/in.}^3$$

Material	*Density*		*Specific gravity*
	lb/in.^{3}	*lb/ft.*^{3}	
Mild steel	0·28	480	7·84
Copper	0·32	545	8·95
Aluminium	0·095	162	2·56
Zinc	0·25	445	7·14
Tin	0·26	452	7·28
Lead	0·41	708	11·36
Brass (average)	0·3	503	8·33

Specific Gravity. The specific gravity of a material is the ratio of weights of unit volumes of the material and water. The density of water is 0·036 lb per cubic inch. The specific gravity of a material can therefore be found from

$$\text{Specific gravity} = \frac{\text{Density of material}}{0{\cdot}036}$$

Accurate Measuring Equipment

The Rule. When a dimension is given on a drawing as a vulgar fraction, it may be assumed that a rule may be used for the measurement. The degree of accuracy to which work may be produced when measurements are made by a rule depends on the quality of the rule and the skill of the user. On a good quality rule the marks vary from 0·005 in. to 0·007 in. wide, so a degree of accuracy much closer than ± 0·008 in. cannot be expected; a good workman, however, may be able to work to closer limits than this.

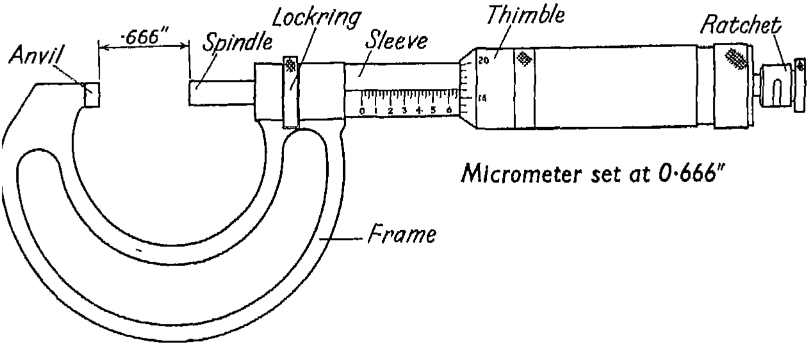
Sheet metal workers' rules are usually made of stainless steel or brass; tempered steel is not a suitable material due to the acidified atmospheres associated with soldering stoves and burned fluxes. Rules are 24 in. long and jointed at the middle by a rivet, which allows the rule to fold upon itself and become 12 in. long when not in use. Each inch is divided into $\frac{1}{16}$ths, and occasionally into $\frac{1}{64}$ths, while special rules are available to give the circumference of any circle up to 12 in. in diameter without resorting to calculation.

Another special feature of this type of rule is a *line of chords* on which a scale is made to represent all angles between 0° and 120°. Dividers are used to measure off the length required to represent any desired angle, and this distance is used for setting the chordal length between two small indentations on each leg of the rule. When this has been done the angle between the legs will be found to be the one desired.

The Micrometer

For measurements beyond the accuracy of the rule the micrometer is commonly used and can be read directly to $\frac{1}{1000}$ in.

Fig. 83 shows a standard micrometer and its important component parts. The spindle screws into the barrel and is attached to the



Micrometer set at 0·666"

FIG. 83. Micrometer

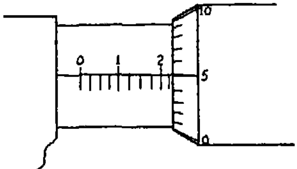
thimble at its inner end. The frame is of forged steel or cast iron and is designed to be rigid and withstand any tendency to 'give' under normal use. The inserted anvil and the spindle faces are hardened, ground and lapped perfectly flat. The spring-loaded ratchet is used to avoid any error due to difference of 'feel', as it will always slip under the same load.

The spindle and sleeve are screwed 40 threads per inch. When therefore the thimble is turned through one revolution the spindle moves $\frac{1}{40}$ in. or $\frac{1}{40} \times \frac{25}{25} = \frac{25}{1000}$ in., or '25 thou'.

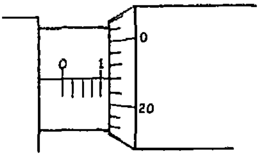
The thimble end is graduated into 25 equal divisions. If it is turned from one graduation to the next it has moved through $\frac{1}{25}$ of a revolution which will move the spindle $\frac{1}{25}$ of $\frac{25}{1000} = \frac{1}{1000}$ in.

The sleeve is graduated along its length for one inch into tenths and fortieths. One turn of the thimble will move its edge $\frac{1}{40}$ in., $\frac{25}{1000}$ in., along the sleeve, or one of the sleeve graduations.

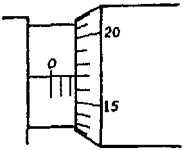
Reading the Micrometer. With a 0–1 in. micrometer, when the zero on the thimble coincides with the zero on the sleeve the anvil and spindle just touch. If now the spindle is moved away from the anvil the distance between them is read off from the sleeve and thimble.



(a) Reading 0·230"



(b) Reading 0·122"



(c) Reading 0·067"

FIG. 84. Micrometer readings

Consider the micrometer setting shown in Fig. 83. The sleeve shows six-tenths and two-fortieths, the thimble sixteen divisions. The reading is then:

$$\frac{6}{10}\text{ in.} + \frac{2}{40}\text{ in.} + \frac{16}{1000}\text{ in. or } \frac{6}{10}\text{ in.} + \frac{50}{1000}\text{ in.} + \frac{16}{1000}\text{ in.}$$

in decimals: $0{\cdot}6\text{ in.} + 0{\cdot}05\text{ in.} + 0{\cdot}016\text{ in.} = \underline{0{\cdot}666}\text{ in.}$

Probably the best way to read the micrometer is to call the tenths, 'one hundred thou', and the fortieths, 'twenty-five thou'.

Various micrometer settings are shown in Fig. 84.

The Vernier

It is now known that when using a rule great accuracy cannot be expected. The vernier scale is a device which allows micrometer accuracy to be read from a rule-like scale. In principle the vernier is made up of two scales, a long fixed scale and a short sliding scale. The long scale is a rule which has its inches divided into tenths and the tenths into fortieths. The sliding scale has a length of six-tenths of an inch divided into twenty-five.

On the fixed scale each small division = $\frac{1}{40}$ in. = 0·025 in.

On the vernier scale each small division = $\frac{1}{25}$ of $\frac{6}{10}$ in. = $\frac{6}{250}$ in. = 0·024 in.

The difference therefore between a small division on the fixed scale and a small division on the vernier scale is:

$$0{\cdot}025 \text{ in.} - 0{\cdot}024 \text{ in.} = 0{\cdot}001 \text{ in. (one thou).}$$

The Vernier Caliper is used for external and internal measuring. It has the advantage of a large range (over 6 in. or 12 in. depending on the size), and fine accuracy. It is not however as accurate in use as the micrometer as the long blade makes it out-of-balance and prohibits the fine ‘ feel ’ that can be obtained with the micrometer. It is also rather more difficult to read than the micrometer.

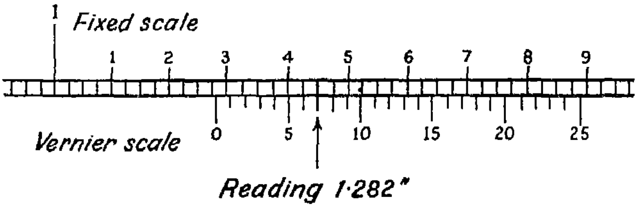


FIG. 85. Vernier scale

Reading the vernier (Fig. 85). When the zeros on both scales coincide the jaws are closed. If we open the jaws until the first line on the vernier scale coincides with the first on the fixed scale the jaws will be now ‘ one thou ’ apart (since the graduations are 24 and 25 thou respectively). When the second vernier line meets the second fixed scale line the jaws will be 2 thou apart, and so on. In reading the vernier, then, the zero on the vernier scale is the first datum line. Up to this zero the inches, tenths, and fortieths on the fixed scale are read off. Now the vernier scale is inspected for a line which is coincident with a fixed scale line. The number of such a line is the number of thousandths to be added to the fixed scale reading.

The Vernier Height Gauge (Fig. 86) is used for inspection and accurate marking out of work on the surface table. It is in effect a surface gauge incorporating a vernier scale. It is read in the same way as the vernier caliper and the scribing point can be quickly set to a ‘ thou ’.

Heat

Heat is a form of energy obtained from: (1) the sun, (2) chemical action, (3) mechanical action, friction, (4) electrical action, an electric fire for example, (5) atomic fission.

Effects of Heat

When a body is heated various changes may take place: (*a*) it expands, (*b*) it may change in composition and physical state, (*c*) its colour alters, (*d*) its temperature rises.

Temperature is the term used to indicate the degree of hotness of a body and is measured by a thermometer which is an instrument for comparing temperatures. There are two types of thermometer commonly used, the *Centigrade* and the *Fahrenheit*. The Centigrade thermometer is in most common use and on this the freezing point of water is marked zero degrees (0°) and the boiling point of water 100 degrees (100°). Between these two points are 100 equal divisions.

The same points on the Fahrenheit thermometer are marked 32° and 212° respectively (the degree sign is °).

100 C divisions = 180 F divisions.

$\therefore$ 1 C division $= \frac{180}{100} = \frac{9}{5}$ F divisions.

Since there is a difference of 32 divisions at freezing point, to convert the scales:

$$°C = \frac{5}{9}(°F - 32)$$

$$°F = \frac{9}{5}°C + 32$$

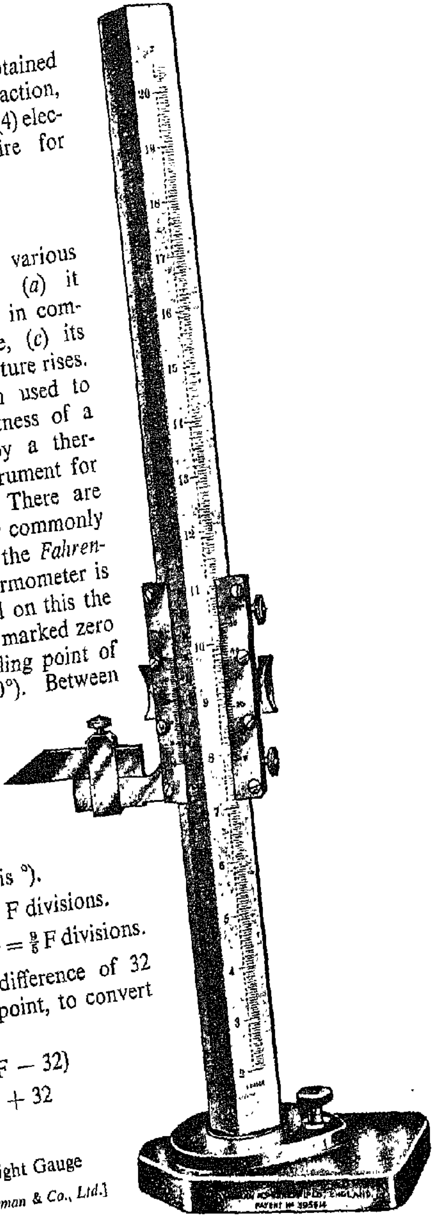


FIG. 86. Vernier Height Gauge

[*James Chesterman & Co., Ltd.*]

EXAMPLE. *Convert* 50°F to °C.

$$\begin{aligned} °C &= \tfrac{5}{9}(°F - 32) \\ &= \tfrac{5}{9}(50 - 32) \\ &= \tfrac{5}{9} \times 18 = \underline{10°C.} \end{aligned}$$

Convert 75°C *to* °F

$$\begin{aligned} °F &= \tfrac{9}{5}°C + 32 \\ &= \tfrac{9}{5} \times 75 + 32 \\ &= 135 + 32 = \underline{167°F.} \end{aligned}$$

States of Matter

In order to understand fully the various states of matter, the meaning of the words atom and molecule should be explained first.

Imagine that a small cube of metal is cut into several smaller parts, and each of these parts is again cut in the same way. If this process were continued until the parts were too small for any further division, then the smallest of all the parts would be termed atoms. These are the ' bricks ' of which all metals are made, and they have a definite attraction for each other.

Atoms are grouped by nature into molecules, and a molecule may contain one, two or more atoms. A molecule is the smallest particle of a substance that can exist separately. These molecules in a solid are very closely packed—the closer they are packed the heavier is the solid—but even in this state they are in continuous vibration although they cannot move about. The very strong attraction between each molecule is the reason why a metal resists any attempt to change its shape when it is worked at normal temperatures.

Suppose now, the metal is heated. The molecules become ' excited ' and the vibration intensified, thus reducing the attraction between them and causing the metal to expand. If heating is continued the expansion will enable the molecules to become detached from each other, leaving them free to move about, and at this point the metal changes from a solid to a liquid. The temperature at which this change occurs is known as the melting point and varies for each metal.

Should heating be continued beyond the melting point, the molecules will become so excited that they will lose all form of attraction for each other and be free to move in any direction by themselves. At this point the liquid becomes a gas and has no definite shape.

It can be seen therefore that the three states of matter—*solid*, *liquid* and *gaseous*—are closely related to each other and that by giving or taking away heat from a metal it can be changed from one state to another.

Physical Changes. Suppose a block of ice is heated until it becomes water, which is then boiled until it becomes steam. It can be said that a solid has been converted into a liquid and then into a gas by simply heating it. If the steam is not allowed to escape and is cooled down it will become water again, and on further cooling it will become ice, thus reverting to its original state. Furthermore, it will be found that no loss of weight will occur in the process. These changes are known as physical changes and may take place in many ways.

Physical changes may be utilised to aid the sheet metal worker. For instance, when a soldered or brazed joint is required, the jointing metal is heated until it becomes liquid, allowed to flow over the joint and then cooled. Similarly, when a metal becomes work-hardened due to hollowing, raising, or bending, it may be restored to its original state by the process of annealing. These are all physical changes.

Chemical Changes. Chemical changes are permanent changes because the substance is usually altered, heat is liberated or absorbed and a new substance is formed. It is rarely possible for the substance to revert to its original state.

If a piece of copper is dipped into a nitric acid bath a violent chemical change will take place and the metal will dissolve in the acid. The same will occur when zinc and hydrochloric acid are in contact with each other, leaving a substance known as zinc chloride which is used as a soldering flux.

It is possible to generate electricity by means of chemical changes.

Elements. An element is a simple substance which cannot be split up into anything simpler. It is made up of only one kind of molecule. All pure metals are elements, for example, aluminium, lead, tin, iron (not steel), copper and zinc.

Mixtures. When two or more elements, or compounds, are mixed together mechanically, i.e., without causing any chemical reaction, this is known as a mixture. For example, if salt and sand were mixed the salt could be removed by dissolving it in water which would leave behind the sand. The salt could then be recovered by boiling off the water. Since the salt and sand would not react with each other this would be a mixture, the salt and sand would each retain its original form and physical properties.

Compounds. A compound is formed by the combination of two or more elements which have reacted with each other to form a substance in all respects quite different from the elements from which is made. Water, carbon dioxide, zinc chloride and rust are all examples of chemical compounds.

Corrosion—The Effect of Air and Water

Rusting, or the corrosion of metals, is important to every metal worker. It is reckoned that it costs Britain over £600 million each year to prevent or remove rust.

Corrosion is a chemical electrolytic process and a full treatment of it is beyond the scope of this book, in which it will be considered in a very elementary way.

Before an iron or steel will rust there must be present both air and water; either alone will not cause it. Dry air or water containing no air (distilled water for example) will not cause rusting. Rust is iron oxide, that is, iron combined with oxygen. Simply then, rust may be considered as due to oxygen from the air attacking the iron and combining with it to form an oxide. It is however more accurately described as an electrochemical action, somewhat similar to the decomposition of a dry cell torch battery, which explains why both air and water must be present. Water containing no air is electrically neutral and therefore presents no conductor of electricity, nor of course does dry air. Once rusting starts it is self-generating, that is, if the supply of air and water is cut off the process will continue, because of the electrochemical decomposition. The *rate* of rusting slows down as the rust deepens.

In most towns today metals have to withstand much more than pure air and water. The atmosphere is polluted with waste smoke and gases, most of which are chemically active. These accelerate rusting considerably and make the prevention problem more acute. The same applies to marine installations where highly corrosive salt water spray is encountered.

It is obvious then that rust prevention is necessary. It is possible, for example, to choose the right metal for a given atmosphere, to reduce the humidity of the workshop, or apply a protective coating of paint or use some other method such as chromium plating, galvanising, tinning, parkerising, etc.

With many metals the first film of oxide formed seals the underlying metal from further attack. Aluminium, copper, the brasses and bronzes, and stainless steel all resist corrosion in this way.

An effect which is similar to rusting is the scaling of iron or steel when it is heated. This scale is pure iron oxide and is due to the heat accelerating the oxygen attack. Obviously no moisture is present and the process is one of pure chemical action.

$$\text{Iron} + \text{oxygen} = \text{iron oxide (scale).}$$

MATERIALS

Iron and Steel

The Latin word for iron is ' ferrum ', hence iron, steel and cast iron are known as ' ferrous metals ', distinguishing them from the other ' non-ferrous ' metals. Now 100 per cent. pure iron has not yet been obtained, although a metal of greater than 99·995 per cent. purity has been prepared, but this is not used commercially.

Wrought iron may be regarded as almost pure iron, although it may contain between 0·5–1·0% impurities. Steel may contain up to 1·5% carbon, but the grade of steel most used in the sheet metal trades contains about 0·2% carbon. Other elements besides carbon are always present in steel; some, like manganese, are beneficial; others, for example sulphur, are detrimental, and steel makers endeavour to reduce the amount of harmful impurities to a minimum.

Cast iron contains 5–7% of total impurities, including carbon, silicon, manganese, phosphorus and sulphur. The following data illustrate some of the mechanical properties of irons and steels containing varying amounts of carbon; no attempt will be made at this stage to show the effect of the other impurities.

Wrought Iron

Composition—Iron plus 0·02–0·1% carbon.

Tensile strength—22 ton/sq. in.

Comparatively soft—Easily scratched or indented.

Very tough—It can be repeatedly bent to and fro or twisted without fracture.

Malleable—It may be rolled into thin sheets without cracking and also beaten into shapes.

Ductile—It can be drawn into wire.

Wrought iron is easily welded by hammering together two white-hot pieces. Welding by gas or arc is difficult due to a pasty condition prevailing at the melting point, which also prevents wrought iron being used in the formation of castings.

Uses—Rivets, chains, hooks, draw bars, sheets, and ornamental work.

Mild Steel

Composition—Iron plus 0·15–0·3% carbon.

Tensile strength—28 ton/sq. in.

The metal is somewhat harder than wrought iron, though *toughness*, *malleability* and *ductility* are less. Mild steel is *easily welded* by all known methods without resorting to the aid of fluxes or special filler rods. Mild steel is *difficult to cast* and *does not harden* when quenched from a red heat, although quenching is not recommended as a method of cooling after annealing.

Uses—Rivets, sheets, plates, unstressed nuts and bolts, forgings, etc.

Medium Carbon Steel

Composition—Iron plus 0·3–0·6% carbon.

Tensile strength—40 ton/sq. in.

The increase in carbon content increases the hardness, though this is accompanied with some brittleness, and further reduces toughness, malleability and ductility. *Welding is difficult* and requires special rods and fluxes or, alternatively, bronze welding is carried out. Medium carbon steel will harden somewhat if water-cooled from a red heat.

Uses—Hammer heads, stakes, small tools, tubes, wire ropes and slings.

High Carbon Steel

Composition—Iron plus 0·6–1·2% carbon.

Tensile strength—Over 40 ton/sq. in.

High carbon steel is hard and strong with only a *little ductility* and *malleability*. It is more brittle than any of the other carbon steels. The metal is used mainly in the production of cutting tools and machinery such as guillotine blades, files, drills and cold sets, and it is in these forms that the metal is employed in sheet metal work industries.

Cast Iron

Composition—Iron plus 2·5–4·5% carbon.

Tensile strength—7·3 ton/sq. in.

This metal is in many respects quite unlike steel. It is weak and brittle with a hard outer skin. Cast iron cannot be worked, but only cast, and therefore is used in the form of machine beds, levelling blocks and small castings of all kinds.

Rolled Steel Sheets

Although metals were rolled into sheets as long ago as the early 16th century, the process was only used extensively in the late 18th century, when much of the preliminary investigations were carried out by Henry Cort. Nowadays mild steel sheets containing about 0·1% carbon are probably the commonest and cheapest of materials handled by the sheet metal worker, besides forming the base material for coated sheets, such as tinplate, terneplate and galvanised iron.

Hot rolling. The steel arrives in the form of flat bars about ½–1 in. thick, and it is cut into lengths equal to the desired width of the finished sheet. These bars are heated in a furnace to a white heat and rolled edgeways through power-driven steel rolls until the thickness has been reduced to about 0·15–0·2 in. After hot rolling the sheets are passed through another furnace which is divided into two sections. The first section is utilised to anneal the metal and the second section for rapid cooling, which is carried out by passing water through a steel jacket enclosing the second compartment. The water does not touch the sheets of steel, but the heat radiated from them is carried away by a continuous flow of cold water, and the metal quickly cools.

During the hot rolling and subsequent annealing the surfaces of the sheets become oxidised and therefore have to be 'pickled' to make them perfectly clean. The pickling is carried out in a lead-lined bath of warm dilute sulphuric acid, 1 part acid to 20 parts water, in which the sheets are immersed for about 5–10 minutes until all the oxides have been removed. They are then washed in warm water to remove the acid.

Cold rolling. The sheets are now transferred to the cold-rolling mill where electric motors, working through a gear train, drive steel rolls ground accurately to a slight camber, to allow for the deflection of the rolls by the high pressures. The plates are passed backwards and forwards through the rolls until the correct thickness is obtained. This is gauged by measurement of the length of the sheet. When the sheets have been rolled to the desired thickness they are cut to correct sizes by a power-driven guillotine.

Close annealing. At this stage the metal is in a strained condition due to the heavy rolling. To relieve this, the sheets are stacked on a fire-brick and sand base, then totally enclosed with further bricks fluted with fire-clay. A steel outer jacket is then placed in position and to it are connected gas jets. In this way it is possible to heat the sheets and anneal them without contact with the air, so preventing any oxide being formed on their surfaces. This is a slow process of many days, but as the furnace contains several hundreds of sheets the ultimate results of an oxide-free annealed sheet fully justify the time spent.

Patent flattening. The sheets, after cooling, are often slightly buckled due to the prolonged heating received in the furnace. To remove the buckles the plates are placed in a machine which grips the ends and slightly elongates them, producing perfectly level sheets.

Boxes of steel sheets which have been cold rolled, close annealed and patent flattened are usually marked C.R., C.A., P.F.

Metal Gauges

Gauge No.	*IMPERIAL s.w.g.*		*BIRMINGHAM*		*AMERICAN*	
	in.	*mm*	*in.*	*mm*	*in.*	*mm*
1	·300	7·620	·300	7·620	·2893	7·348
2	·276	7·010	·284	7·213	·2576	6·543
3	·252	6·400	·259	6·578	·2294	5·827
4	·232	5·892	·238	6·045	·2043	5·189
5	·212	5·384	·220	5·588	·1819	4·621
6	·192	4·876	·203	5·156	·1620	4·115
7	·176	4·470	·180	4·572	·1443	3·664
8	·160	4·064	·165	4·190	·1285	3·263
9	·144	3·657	·148	3·759	·1144	2·906
10	·128	3·251	·134	3·403	·1019	2·588
11	·116	2·946	·120	3·048	·0907	2·304
12	·104	2·640	·109	2·768	·0808	2·052
13	·092	2·336	·095	2·413	·0719	1·827
14	·080	2·032	·083	2·108	·0641	1·627
15	·072	1·828	·072	1·828	·0570	1·449
16	·064	1·625	·065	1·651	·0508	1·290
17	·056	1·422	·058	1·473	·0452	1·149
18	·048	1·219	·049	1·244	·0403	1·024
19	·040	1·016	·042	1·066	·0359	0·911
20	·036	0·914	·035	0·889	·0319	0·811
21	·032	0·812	·032	0·812	·02846	0·722
22	·028	0·711	·028	0·711	·02535	0·643
23	·024	0·609	·025	0·635	·02257	0·573
24	·022	0·558	·022	0·558	·02010	0·511
25	·020	0·508	·020	0·508	·01790	0·454
26	·018	0·457	·018	0·457	·01594	0·404
27	·0164	0·416	·016	0·406	·01419	0·360
28	·0148	0·375	·014	0·355	·01264	0·321
29	·0136	0·345	·013	0·330	·01125	0·285
30	·0124	0·314	·012	0·304	·01002	0·254
31	·0116	0·294	·010	0·254	·00892	0·226
32	·0108	0·274	·009	0·228	·00795	0·201
33	·0100	0·254	·008	0·203	·00708	0·180
34	·0092	0·233	·007	0·177	·00630	0·160
35	·0084	0·213	·005	0·127	·00561	0·142
36	·0076	0·193	·004	0·101	·00500	0·127

Weights of mild steel sheets per sq. ft

1 cu. in. of mild steel weighs 0·286 lb

Imperial s.w.g.	10	12	14	16	18	20	22	24	26	28
Lb per sq. ft	5·27	4·28	3·3	2·64	1·98	1·47	1·15	0·905	0·74	0·61

Alloys

It has been shown that the mechanical properties of iron are altered by the addition of carbon. So also are the properties of any metal altered by alloying other elements with it. Alloying is generally carried out by mixing two or more molten metals, although sometimes the ores themselves are mixed before melting.

Alloys are formed to produce:

(*a*) An increase in tensile strength, hardness, etc.; e.g., from iron to steel.
(*b*) A reduction in melting point; a lead/tin solder of 62% tin and 38% lead melts at 183°C, but lead melts at 327°C and tin at 232°C.
(*c*) A change of colour; the redness of copper can be changed to a range of golden yellow colours by adding zinc to make the brasses.
(*d*) A metal which will respond to heat treatment; e.g., from iron to high carbon steel.
(*e*) Greater resistance to corrosion; e.g., adding 18% chromium and 10% nickel to steel to produce stainless steel.
(*f*) Sounder castings; pure copper is difficult to cast—by adding zinc sound castings are readily produced.
(*g*) Better machining qualities; 1–3% lead added to steel produces ‘ free cutting steel ’.

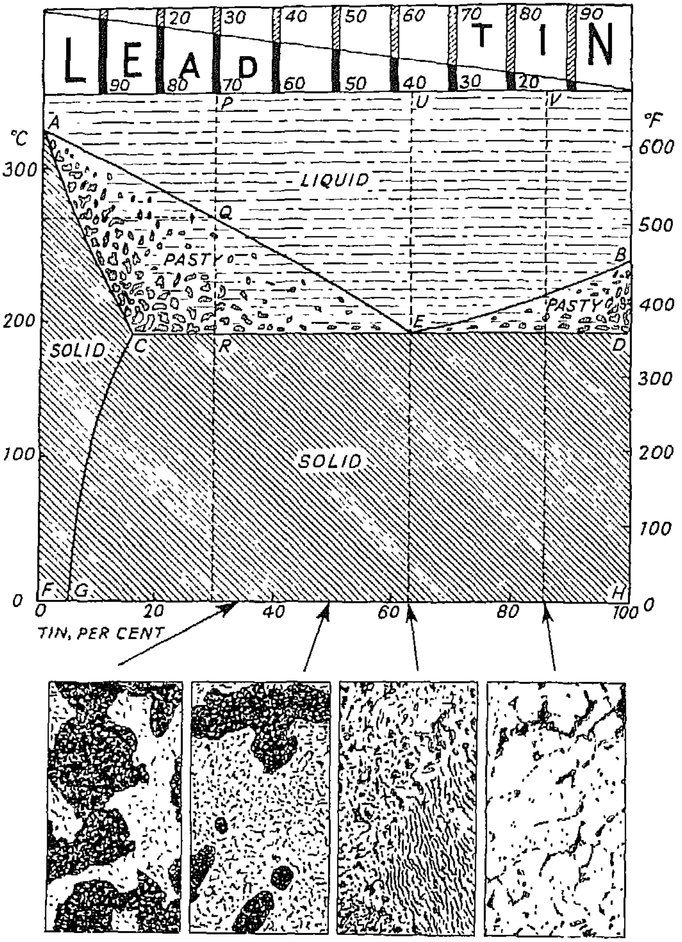
These, often dramatic, changes are produced because generally when two metals are alloyed they form what is known as a solid solution of one in the other; that is, the two metals combine to form another entirely different metal and neither of the original two can be distinguished under the most powerful microscope. It is this solid solution which produces the effects.

Those alloys used in sheet metal work will now be described.

Constitution of Solders

The following extract on the composition of solders has been reproduced by kind permission of the Tin Research Institute from a paper by L. G. Earle:

> The composition of a solder is defined in percentages of tin, lead, antimony and impurities, while the physical structure and its behaviour as regards melting and solidifying are indicated by the diagram (Fig. 87(a)). The entire range of tin-lead solders is depicted with pure lead on the extreme left and pure tin on the extreme right, while temperature is represented by the height above the base line. Pure lead is seen to have a melting point of



(a) 35 per cent tin (b) 50 per cent tin (c) 63 per cent tin (d) 85 per cent tin

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FIG. 87(a). Constitutional diagram of the Tin–Lead Alloys.

Photomicrographs of various solders. The black areas in (*a*) and (*b*) show lead embedded in eutectic solder; (*c*) is entirely eutectic and (*d*) shows eutectic as darker material embedded in pure tin.

327°C, and pure tin of 232°C. Mixtures in all proportions have their melting points indicated by the lines AE and EB. E is a point of cardinal interest because it is evidently the easiest melting of all the tin-lead solders, and for this reason is known as the eutectic solder. It contains 63 per cent. of tin, remainder lead, and melts at 183°C. This melting point is characteristic of this mixture only of these pure metals, and can be used as a means of identifying this composition, provided that appreciable amounts of antimony and impurities are absent. The photomicrograph (C) shows this characteristic laminated or banded eutectic material. Another peculiarity about the eutectic mixture is obvious from the diagram for it is the only composition which transforms from the liquid to the solid state without passing through an intermediate pasty state. At 182°C eutectic solder is completely solid but at 184°C it is entirely liquid. At 183°C, however, the eutectic may be either liquid or solid or a mixture of both states in any proportion. The change from the solid to the liquid state absorbs heat energy and the reverse change releases that same energy. This phenomenon is the cause of the break in the curve showing the rate of heating or cooling of a mass of eutectic solder (Fig. 87(b)). Time elapses without change of temperature during the transition from one state to the other. The time of transition in the case of a drop of solder is not appreciable but may be several minutes when melting or cooling a ladleful.

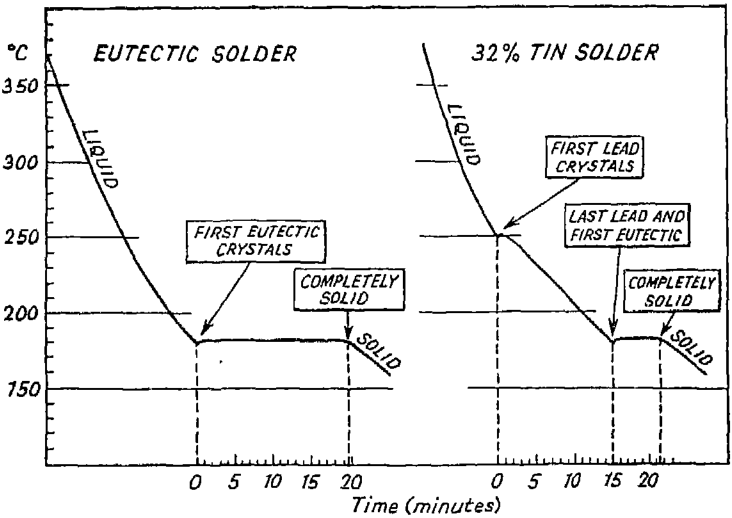

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FIG. 87(b). Typical cooling curves of eutectic and 32% tin solders (L. G. Earle)

The diagram shows that tin-lead alloys containing 20 to 97 per cent. of tin pass from the solid state to a pasty condition when they are heated to a temperature of 183°C. Evidently they all

The properties of tin-lead solders

B.S. 219 Grade	Nominal tin content	Limits of tin content		Limits of antimony content		Melting temperatures				Specific gravity	Electrical conductivity (% of copper)	Applications
		Min.	Max.	Min.	Max.	Solidus	Liquidus	Solidus	Liquidus			
	%	%	%	%	%	°C	°C	°F	°F			
—	100	—	—	—	—	232	232	450	450	7·3	13·9	Pre-tinning; special instrument work
E	95	94·5	95·5	—	0·5	183	223	361	434	7·5	13·7	Electrical instruments
A	65	64	66	—	1·0	183	186	361	367	8·4	11·9	High strength; quick setting
	63	—	—	—	—	183	183	361	361	8·4	11·8	No plastic range; lowest melting point
K	60	59	61	—	0·5	183	189	361	372	8·5	11·6	Fine electrical and tinsmiths' work; quick setting
F	50	49	51	—	0·5	183	214	361	417	8·9	10·9	General purposes; tinsmiths' and machine soldering; non-antimonial
B	50	49	51	2·5	3·0	185	203	365	398	8·7	9·8	General purposes: where an antimonial solder can be used
—	45	—	—	—	—	183	225	361	437	9·1	10·5	General purposes; non-antimonial
M	45	44	46	2·3	2·7	185	215	365	419	8·9	9·5	General purposes; antimonial
G	42	41	43	—	0·4	183	232	361	450	9·2	10·3	General purposes, dipping galvanised work
—	40	—	—	—	—	183	236	361	457	9·3	10·1	Can soldering
C	40	39	41	2·0	2·4	185	228	365	443	9·1	9·2	General purposes but slow setting
H	35	34	36	—	0·3	183	246	361	475	9·5	9·6	Non-antimonial for wiping joints on cables
L	32	31	33	1·6	1·9	185	245	365	474	9·5	8·8	Post Office cable wiping solder
J	30	29	31	—	0·3	183	255	361	491	9·7	9·3	Non-antimonial; wiped joints and dipping baths
D	30	29	31	1·0	1·7	185	248	365	478	9·6	8·6	Plumbers' wiped joints
N	*18·5*	*18*	*18·5*	*0·75*	*1·0*	*185*	*275*	*365*	*527*	*10·2*	*8·1*	*War-time dipping solder*
	Lead	0	—	—	—	327	327	621	621	11·4	7·9	Pure lead

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contain some eutectic as well as a proportion—characteristic for each solder—of higher melting constituents; for example, the ratio of eutectic material to higher melting constituents in plumbers' solder is the ratio of the distance RC to the distance RE. In other words, the eutectic proportion of a solder rises to 100 per cent. at E and falls away in direct proportion until at C (and D) there is none present.

The condition of plumbers' solder during the wiping of a joint can be deduced from the diagram. While it cools to 270°C it is completely liquid and then crystals of lead begin to separate until the residual liquid has become so enriched in tin that it is of the eutectic composition and begins to crystallise as such at 183°C; in doing so it cements together the earlier formed lead crystals (see the photomicrograph (*a*) which shows straggling dark masses of lead set in the lighter background of eutectic). It might be supposed that a solder with the composition corresponding to C on the diagram, and therefore having the longest plastic range, might be more easily manipulated than the 30% tin solder, but the lack of sufficient liquid eutectic gives rise to voids and porosity between the lead crystals.

The presence of eutectic in all the ordinary tin-lead solders means that solders are no longer truly solid at 183°C or higher, even although they may not be obviously pasty. In the pasty range they are all devoid of strength and even at thirty or forty degrees centigrade below the eutectic melting point, solders are only about half as strong as at room temperature, so it is necessary to avoid straining soldered joints before they have had time to cool.

It is clear from the diagram that, in practice, eutectic solder (63% tin, 37% lead) melts most quickly and sets solid most quickly and it is, therefore, the fastest solder to work with.

The Brasses

Brass is an alloy of copper and zinc containing from 55 to 95% copper, but an exception is brazing spelter, which contains 50% of each. All brasses are highly corrosion resistant.

There are two groups of brasses, cold working and hot working.

Cold working brasses contain not less than 63% copper, are very malleable and ductile and can be pressed, spun, rolled and drawn. Cold working hardens and strengthens the brass but makes it more brittle. If a great deal of cold working has to be done, the brass must be annealed, or softened, at some intermediate stage to prevent cracking. Annealing is done by heating to about 600°C, soaking at this temperature until the heat is even throughout and then allowing to cool in air. Brasses having more than 70% copper can be quenched in water to save time; this also cleans off the oxide.

If the annealing temperature is too high the metal becomes over-annealed. The crystals grow over large, which makes the metal weak and if it is now cold worked an ' orange peel ' effect appears. Large crystals appear on the surface and in severe cases these can actually be picked out.

In severely cold worked brass, for example a deep pressing, the internal stresses are usually unevenly distributed. If such a part is left in a corrosive atmosphere, especially if ammonia is present, cracking takes place, called ' season cracking '. These stresses can be relieved by a low temperature heat treatment, heating to 250° to 270°C. The part will now not be subject to season cracking, although it is not annealed. The brasses having more than 85% copper are not usually liable to season cracking.

Hot working brasses have about 58 to 60% copper. They are harder and stronger than the cold working group and are very plastic over a wide range of temperature. Hot working, such as hot rolling, extruding and stamping, is therefore readily carried out. A certain amount of cold working can also be done on these without cracking.

Some widely used Brasses

Gilding metals, 80 to 95% copper. In colour they range from yellow to golden, and because of this and the ease with which they can be worked they are used for jewellery, architectural ornaments, etc. For engineering purposes they are usually supplied as cold rolled sheet or strip.

Cartridge brass, 70% copper, 30% zinc. This is used for deep drawn parts, such as cartridge and shell cases, headlamp casings, cooking utensils etc., where very high ductility and strength are required. They also have a fine pale yellow colour and are used for household ornaments. They are supplied as cold rolled sheet and strip, and also as tube.

Yellow metal or **muntz metal**, 60% copper, 40% zinc. This is suitable for only a small amount of cold work, such as hand work. It is largely used for extrusion, casting and hot stamping. It is supplied as sheet, but since this is normally hot rolled the surface is rough compared with the fine cold rolled finish. It is also supplied as rod and extruded sections.

Brazing spelter, 50% copper, 50% zinc. This common spelter is varied sometimes by the addition of 10–15% silver, which makes ' silver solder '. It is expensive but its high strength has many uses. High-grade silver solders containing up to 50% silver are used by the silversmith.

Temper of Brass

Cold working increases the tensile strength and hardness of brass while lowering its ductility. By varying the amount of cold work done on it, brass sheet and strip can be supplied in various tempers.

Soft, fully annealed, brass is progressively rolled to produce quarter hard, half hard, hard, spring hard and extra hard tempers. These are especially useful for press work, where the temper is chosen to produce the correct hardness in the finished article after working.

Bronze is an alloy of copper and tin, although it may contain some zinc, phosphorus, nickel or lead. The majority of bronzes are casting metals and are therefore of little direct use in sheet metal work.

Cold working bronzes contain up to about 8% tin. These, like the cold working brass, can be cold rolled or drawn. Also, like cold working brass, they are hardened and strengthened by cold work and are supplied as sheet or strip in various tempers.

Strength and weight of materials

	Weight of a cubic inch	*Ultimate* tensile stress per sq. inch
Aluminium sheet	0·096 lb	12 tons
„ cast	0·092	8
Copper bolts	0·318	17
„ cast	0·31	8·4
„ sheet	0·316	13·4
„ wire	0·32	26
Iron, cast from	0·252	6
„ „ to	0·273	13
„ „ average	0·26	7·3
„ wrought from	0·273	16
„ „ to	0·281	29
„ „ average	0·28	22
„ wire	—	40
Lead cast	0·408	0·8
„ sheet	0·41	1·5
Platinum sheet	0·828	—
Steel, mild	0·286	25·35
Tin, cast	0·262	2·0
Zinc, „	0·252	3·3
Alloys		
Aluminium bronze 90 to 95 per cent, copper	0·276	32
Brass, cast	0·3	8
„ sheet	0·301	14
„ wire	0·307	22
„ 3 copper 1 zinc	0·3	13·1
„ 2 „ 1 „	0·299	12·5
„ 1 „ 1 „	0·296	9·2
Gun-metal, 10 copper, 1 tin	0·306	16·1
Speculum metal	0·264	3·1
White metal (Babbitt)	0·263	—

The addition of up to 0·4% phosphorus increases the strength and hardness and produces **cold working phosphor bronze.** This is used for making chemical storage vats, stills and for general coppersmithing. The ultimate tensile strength is about 30 tons per sq. in.

Cold working aluminium bronze *is an alloy of copper and aluminium* containing about 4 to 7% aluminium. Cold working hardens and strengthens it. An oxide film quickly forms on the surface, which acts as a protective layer and prevents further oxidisation and tarnishing. Because of the fine golden colour and lasting finish, aluminium bronze is used for making jewellery, cigarette cases, plaques, trays, etc. Tensile strength is about 25 tons per sq. in.

Cold working silicon bronze, a typical work hardening alloy, contains 3% silicon, 1% manganese, 86% copper. This has high resistance to corrosion and is supplied in sheet for making storage tanks, pickling baths, cisterns for marine work and press work articles. Tensile strength is about 23 tons per sq. in.

Galvanised Sheets

Galvanising is the application of a thin coating of zinc to iron or steel sheets, from which all the scale has been removed by pickling in a solution of hydrochloric acid and water.

Pickling. Stone or lead-lined tanks are used for holding the diluted acid; over-pickling must not take place as this will give the finished sheets a dull, dry appearance. The sheets, when properly cleaned, are washed in water to remove the surplus acid.

Surface coating. The molten zinc, referred to as spelter, is stored *in heated steel tanks divided into two halves. The inlet side holds* a flux box carrying a thick surface layer of sal-ammoniac and tallow. The plates are fed into the flux box and hence, by means of a series of steel rollers, guided below the surface of the molten zinc to the outlet side of the bath, the last pair of rolls being set to apply an even distribution of zinc to the iron. Unlike tinplate, the galvanised sheets are cooled quickly in water and then dried in a heated stove.

A small amount of tin and lead is often added to the zinc to improve the surface texture of the finished plates. Sand and ashes are scattered over the surface of the molten metal and kept moistened with water; this prevents vaporisation losses and reduces the zinc fumes created during the process.

For buckets, baths and other general water-vessels it is common practice to make the articles from uncoated iron or steel sheets and to galvanise after manufacture. The advantages of this are twofold; first, it ensures that the protective coating is not damaged by rolling, jointing or riveting, and secondly the zinc seals the joints and makes the vessels watertight.

In general it is normal to make all small articles from uncoated iron sheets and after pickling to apply the zinc coat by dipping the articles in a bath and hanging them up to dry. Larger work, such as ductwork, guttering and ventilator heads, should be produced from flat plates previously galvanised.

Uses of Galvanised Articles

Iron rapidly oxidises when exposed to damp air or moisture. The oxide formed absorbs further moisture and accelerates the corrosion of the metal; ultimately the whole structure collapses. Zinc also readily oxidises when in contact with air, but the oxide so formed on the metal provides a protective coat or a barrier to further oxidisation. Pure zinc is both costly and unsuitable for many production processes due to its brittleness at normal temperatures, but by the use of galvanised iron sheets it is possible to produce corrosion resisting articles at a low price.

Galvanised corrugated sheets have an extensive application in roofing work. No definite life can be assigned to such a roof until all the conditions are known. The life will depend upon the quality of the iron, the thickness of the zinc coat, and also the surrounding atmosphere.

For instance, the acidic atmosphere of a large manufacturing city will destroy the protective coating at about three times the rate of the purer air found in the open country. Also a galvanised sheet will last much longer in a dry atmosphere than in a damp one.

The life of galvanised water vessels, such as buckets, will be greatly affected by the condition of the water. Soft water attacks zinc and quickly corrodes it, but hard water tends to cover the metal with a coat of lime which gives added protection to the plate.

Zinc coated sheets should never be used in the manufacture of food or consumable fluid containers. The production of such articles is dealt with on page 137.

A pencil or chalk should be used for marking out a galvanised sheet as a scriber will cut through the coating and expose the steel surface, thus accelerating corrosion.

Sherardizing is another process by which a surface film of zinc may be applied to iron. It can be regarded as a galvanising process by which an *even surface* is obtained; for example, screw threads may be treated without altering their size. Articles requiring such treatment are placed in a heated and mechanically revolving drum. In the drum is also placed a large quantity of zinc powder, the powder and the articles being packed in such a way as to ensure that each article is completely surrounded by the powder and does not touch the adjacent ones. On completion of the packing the drum is slowly revolved and the temperature raised to 375°C. This temperature is just below the melting point of zinc, but nevertheless the iron becomes coated with a thin film of zinc. The revolving of the drum prevents the articles from sticking together and also ensures an even coat, which has a dull, matt appearance. Efforts have been made to produce a polish during the process, but normal practice is to polish the articles after sheradizing has been completed.

Brylanizing. Developments in the U.S.A. have led to the electro-galvanising of wire and sheets by a continuous process. The zinc deposit is obtained direct from the ore, thus avoiding the intermediate step of first making the zinc. At the moment it is one of the lesser known processes, but nevertheless one worthy of being mentioned.

The Manufacture of Tinplate

Tinplate is an excellent combination of metals with the strength of steel and the highly polished appearance of tin. If iron or steel alone were used for food containers, the metal would rust, while tin itself is far too soft and expensive a material to be used in large quantities.

Almost half the tin used in engineering industries goes in making tinplate. The thin surface film prevents the steel underneath from being seen, hence some confusion often arises between tin and tinplate.

Most of the tinplate in Britain is now produced in Wales, but some made in Staffordshire and Worcestershire supplies the many Birmingham firms using it. The manufacture of the plates is a fascinating process and one for which Britain has a world-wide reputation.

The plates. Mild steel sheets of low carbon content are used as the base metal; they have usually been previously cold rolled and close annealed. The sheets are inspected first for freedom from oxides and other surface impurities, then stored in tanks of water containing a small amount of hydrochloric acid until the tinning operation.

Fluxing and tinning. Each plate in turn is immersed in a molten-flux bath containing resin, tallow and other greases and kept until the plate reaches the temperature of the flux. It is then transferred to the first tinpot, a large steel tank full of molten tin, and left until warm enough for the metal to adhere to the surface of the steel. The tin is kept liquid by gas jets burning below the bottom of the tank.

An operation known as *sweeping* is now carried out. It consists of laying the plates on a solid bed and brushing the surfaces with hemp mops to ensure that the whole of the sheet has an even surface of tin. On completion the plate is placed vertically in a second tinpot filled with metal of high purity, and it is extracted by passing through steel rollers which make the coating even and squeeze off any surplus liquid tin. The finished tinplate is then stored in racks until solidification of the surface metal takes place. Most sheets of tinplate appear to have one edge with a thick deposit along it. This is caused by the metal running down the surface during solidification and should be removed before using the sheet.

Modern plants carry out the process of tinplating in one continuous operation. Alterations are being made in methods of manufacture of both the steel sheets and of tinplate itself. The electro-deposition of tin is becoming increasingly popular and where very thin coatings are required has already supplanted the dip process.

Uses of tinplate. Tinplate is not used for outdoor purposes or for water containers as it rusts so easily because of surface imperfections which allow moisture to reach the base metal. It is used extensively in the canning industry for food containers because the metal tin, its oxide or its salts are not toxic or poisonous. This enables fruits, vegetables and meats to be kept in ' tin cans ' for long periods.

Dairy farmers have for many years used tinned iron milk churns and measures, but these are now being replaced with stainless steel ones. Domestic articles, such as cake tins, roasting tins and hot plates, and general articles such as cash boxes, vegetable racks and tobacco containers are made from tinplate, sometimes decorated by painting.

Tinplate must never be heat treated during any manufacturing process, as this will destroy the protective film. Soldering of the metal is easily carried out using a flux of zinc chloride and a lead tin solder. Tinplate should never be welded unless special precautions are taken, for the resulting joints will be weak due to the burned tin being absorbed into the steel. When acid fluxes are used on tinplate they should always be removed after use as they are highly corrosive. Tinplate may be polished by rubbing on powdered chalk or whitening and then removing with a soft clean piece of cotton waste.

Terne Plate

The production processes involved in the manufacture of terne plate are very similar to those of tinplate.

The coating is of a lead/tin composition applied by passing the iron or steel sheets first through a flux bath and then through another bath containing the surface alloy. The sheets are guided through steel rollers revolving in the bath and below the surface. The sheets on cooling have a rather dull grey appearance.

Terne plate can be considered a cheap substitute for tinplate,

Dimensions of Tinplate Sheets

Birmingham Gauge	Substance in.	Size in.	No. of Sheets	Nett weight per box lb.
35·7	·0063	20 × 14	112	55
35·7	·0063	28 × 20	112	110
35·0	·0069	20 × 14	112	60
35·0	·0069	28 × 20	112	120
34·2	·0075	20 × 14	112	65
34·2	·0075	28 × 20	112	130
33·9	·0078	30 × 21	112	153
33·7	·0080	20 × 14	112	70
33·7	·0080	28 × 20	112	140
33·1	·0086	20 × 14	112	75
33·1	·0086	28 × 20	112	150
33·0	·0087	30 × 21	112	171
32·5	·0092	20 × 14	112	80
32·5	·0092	28 × 20	112	160
32·3	·0095	30 × 21	112	184
32·0	·0098	20 × 14	112	85
32·0	·0098	28 × 20	112	170
31·8	·0100	18¾ × 14	124	90
31·6	·0103	20 × 14	112	90
31·6	·0103	28 × 20	112	180
31·6	·0103	20 × 10	225	128
31·5	·0104	18¾ × 14	124	95
31·1	·0109	20 × 14	112	95
31·1	·0109	28 × 20	112	190
31·1	·0109	20 × 10	225	136
31.0	·0110	30 × 21	112	214
31·0	·0110	18¾ × 14	124	100
30·6	·0114	20 × 14	112	100
30·6	·0114	28 × 20	112	200
30·6	·0114	20 × 10	225	143
29·9 I.C.	·0124	20 × 14	112	108
29·9 I.C.	·0124	28 × 20	112	216
29·9 I.C.	·0124	30 × 21	112	243
29·9 I.C.	·0124	20 × 10	225	156
29·9 I.C.	·0124	18¾ × 14	124	110
29·0 I.X.L.	·0139	20 × 10	225	174
28·0 I.X.	·0156	20 × 14	112	136
28·0 I.X.	·0156	28 × 20	56	136
28·0 I.X.	·0156	30 × 21	56	153
28·0 I.X.	·0156	18¾ × 14	124	140
28·0 I.X.	·0156	20 × 10	225	195
27·8 D.C.	·0159	25 × 17	50	94
22·8 D.C.	·0159	34 × 25	25	94
27·8 I.X.X.	·0179	20 × 14	112	156
26·8 I.X.X.	·0179	28 × 20	56	156
26·8 I.X.X.	·0179	30 × 21	56	176
26·8 S.D.C.	·0179	22 × 15	100	184
26·8 S.D.S.	·0179	30 × 22	50	184
26·8 S.D.X.	·0201	22 × 15	100	184
25·8 S.D.X.	·0201	30 × 22	50	184
25·8 I.X.X.X.	·0202	20 × 14	112	176
25·8 I.X.X.X.	·0202	28 × 20	56	176
25·8 I.X.X.X.	·0202	30 × 21	56	198
25·6 D.X.	·0206	25 × 17	50	122

Dimensions of Tinplate Sheets—*continued.*

Birmingham Gauge	Substance in.	Size in.	No. of Sheets	Nett weight per box lb.
25·6 D.X.	·0206	34 × 25	25	122
25·9 S.D.X.X.	·0223	22 × 15	100	204
24·9 S.D.X.X.	·0223	30 × 22	50	204
24·8 X.X.X.X.	·0225	20 × 14	112	196
24·8 X.X.X.X.	·0225	28 × 20	56	196
24·8 X.X.X.X.	·0225	30 × 21	56	220
24·3 D.X.X.	·0240	25 × 17	50	142
24·3 D.X.X.	·0240	34 × 25	25	142
24·1 S.D.X.X.X.	·0245	22 × 15	100	224
24·1 S.D.X.X.X.	·0245	30 × 22	50	224
24·4 S.D.X.X.X.X.	·0267	22 × 15	100	244
23·4 S.D.X.X.X.X.	·0267	30 × 22	50	244
23·1 D.X.X.X.	·0274	25 × 17	50	162
23·1 D.X.X.X.	·0274	34 × 25	25	162
23·1 D.X.X.X.X.	·0308	25 × 17	50	182
22·1 D.X.X.X.X.	·0308	34 × 25	25	182
20·4 D.X.X.X.X.X.X.	·0375	25 × 17	50	222
20·4 D.X.X.X.X.X.X.	·0375	34 × 25	25	222

although it has a special application in duct work for carrying hot air and gases. It is also used extensively for oil cans and lamps and for lining crates used for carrying boxes and parcels. Like tinplate, terne plate can be easily soldered, but not welded. Also it should not be heat treated if the surface coat is to be preserved. It is more easily painted as its surface is not so smooth as tinplate, thus enabling the base paint to obtain a firm hold on the metal. The industrial use of terne plate has not yet been fully exploited, for many articles now produced from tinplate could be just as easily made from terne plate at a much lower cost. It is, however, highly toxic and should never be used with foodstuffs.

Copper

Specific gravity 8·95. Melting point 1055°C.

Physical properties. The metal, having a fine red colour, is characterised when pure by extreme toughness. In the cast condition, the comparatively low *tensile strength* of about 10–11 tons per sq. in. can be increased by cold rolling to 20–26 tons per sq. in., or by wire drawing to 30 tons per sq. in.

At normal temperatures copper in the annealed condition is highly malleable and ductile; its low elasticity allows the metal to be beaten into shapes with comparative ease. Cold working, however, work hardens the metal, reduces its malleability and increases its elasticity. For these reasons, periodic annealing should be carried out during hollowing, raising or spinning processes to eliminate the possibility of cracking. When copper is heated its tensile strength is considerably reduced, thereby enabling hot working to be carried out. Care must be taken to ensure that the metal is not overheated, as at temperatures near its melting point the metal absorbs oxygen and becomes *short* (a tendency to crack).

Pure copper is an excellent conductor of both heat and electricity, although impurities greatly impair these qualities. The metal is capable of being polished, but quickly tarnishes when in contact with oxygen. The polish may be retained by painting the metal with clear varnish or lacquer.

Chemical properties. Under normal conditions copper resists weather corrosion and for many centuries it was used for roof covering. When in contact with a damp atmosphere for long periods the surface becomes covered with a pale green film of rust, known as *verdigris* and is highly poisonous. This protects the metal beneath, thus preventing further corrosion. On account of its high resistance to corrosion by most soils, copper piping is now used for underground distribution of water and gas.

Uses of copper. Copper is available for use in the form of sheets in three main grades: (i) tough pitch, (ii) deoxidised, (iii) arsenical.

(*a*) **Ordinary tough pitch copper** is supplied for all types of fabrication. It contains a small percentage of oxygen and some impurities which impair the conductivity of the metal. Nevertheless this grade of copper is entirely satisfactory for general purposes for which high electrical and heat conductivity is not required. This type of copper is not suitable for welding but can be soldered and brazed.

(*b*) **Phosphorus deoxidised copper,** which may contain arsenic, usually contains about 0·04% phosphorus which enables the metal to be welded. This grade is much used in the manufacture of chemical plant, in the form of sheets which require welding in the course of fabrication.

(*c*) **Arsenical copper** has 0·3–0·5% arsenic in solid solution. It may be either *tough pitch* or *deoxidised*, though the latter is more usual; the tensile strength of arsenical copper is somewhat higher than the previous types and moreover has a better resistance to corrosion by certain chemicals. Arsenical copper does not oxidise easily and therefore can be used at high temperatures without fear of collapse. This property renders the metal suitable for boiler linings and fire boxes on locomotives.

The standard size of a sheet of copper is 4 ft by 2 ft and the metal may be ordered by quoting the wire gauge or by weight. For thin material the wire gauge is more often used while for the thicker sheets the weight per sheet is quoted.

Weights of Copper Sheets per square foot

1 cu. in. of copper weighs 0·32 lb.

Imperial S.W.G.	10	12	14	16	18	20	22	24	26	28
Lb per sq. ft	5·9	4·7	3·69	2·96	2·22	1·66	1·29	1·01	0·83	0·684

Aluminium

Specific gravity 2·56. Melting point 658°C.

Physical properties. The metal has a white silvery appearance, is highly malleable and ductile, and extremely light, about one third the weight of steel. One cubic inch of aluminium weighs about 0·1 lb.

In the cast state the metal has a tensile strength of 7 tons per sq. in., which can be increased to 15 tons per sq. in. by rolling into sheets and drawing into wire.

Pure aluminium is a good conductor of both heat and electricity, second to copper among the industrial metals. Impurities and alloys reduce the conductivity, but the values still remain high by comparison with iron and steel. The metal is non-magnetic and hence is used for protective casings of delicate magnetic instruments such as ships' compasses. Aluminium is unstable at high temperatures and does not change colour on heating. Care must be taken, therefore, not to overheat the metal on annealing for no indication that the metal is approaching the melting point can be seen. The practical method of annealing aluminium is described on p. 186.

Strong and extremely light aluminium alloys can be obtained by adding silium, copper, magnesium and zinc during the refining process. Perhaps the most important of these is duralumin, an alloy containing 3·5% copper and 0·5% magnesium which on heat treatment and subsequent age hardening is an extremely strong material.

Chemical properties. As normally produced, the surface of pure aluminium is covered with a thin film of oxide, rendering the metal practically immune to attack and resistant to atmospheric corrosion. The immunity of aluminium and its alloys to chemical action is one of the main reasons for its use in the manufacture of domestic articles and cooking utensils, in which it has now superseded copper.

Certain substances, such as alkalis and caustic soda, will dissolve the protective oxide film so that the metal below is no longer corrosion resistant. Therefore it is not advisable to use the metal in contact with soap powders or soda, which contain alkalis, for these tend to tarnish the metal and will eventually destroy it.

The surface film of oxide can be thickened by special treatment known as *anodising*. It is carried out by placing the metal in a tank containing chromic acid through which an electric current is passed. The film produced is hard and strong, being integral with the metal, and it has outstanding resistance to chemical attack. Anodised aluminium surfaces can be dyed in a wide range of colours which are resistant to light and heat; by special treatment it is possible to reproduce photographs on the metal. By a similar process the aluminium can be electro-brightened to a high degree and in this condition it is used for reflectors.

Uses of aluminium. In the metal trades industries the metal is mainly used in sheet form. The sheets are made by hot rolling wrought aluminium slabs until about $\frac{1}{4}$ in. thick and then further reducing them by cold rolling. The metal is available in three main grades: (*a*) soft annealed; (*b*) half hard; (*c*) hard.

Intermediate grades may be obtained for special purposes. The hardness is obtained by cold rolling and not by the addition of any alloying elements.

Rolling has the effect of increasing the tensile strength of the metal, while annealing will diminish it but increase the elongation. A hard aluminium sheet will have a minimum tensile strength of 11·5 tons per sq. in. and a 3% elongation. The same sheet when fully annealed will have its tensile strength decreased to 6 tons per sq. in. and its elongation increased to 30%.

Aluminium is soft and ductile and is used with success in such operations as hollowing, raising, spinning and pressing. Complex forms can be easily formed with hand or machine tools, particularly when the metal is in soft temper. Only when the cold working is severe is there any need for interstage annealing. This, coupled with its corrosion resisting properties, renders aluminium suitable for fabrication of aircraft parts, motor car components and body work on railway carriages.

Besides being hygienic, the metal is non-toxic and can be used safely for processing vessels and tanks. Vats for brewing, boiling pans and refrigerator linings are established uses for aluminium. Aluminium foil is used for wrapping foodstuffs and tobacco, while aluminium kettles, pans and teapots are now found in every home. In dairy farming aluminium is employed for almost everything, from milk bottle tops to milking machines. Such is the importance of aluminium in the metal trades that it now ranks second to steel in the list of most-used materials. Since the metal was almost unknown and classified as a precious material 100 years ago, it is remarkable how quickly its applications have been adopted by the engineering trades.

Aluminium can be readily welded by both gas and arc methods, using fluxes to destroy the oxide film. These fluxes are highly corrosive and all traces of them must be removed on completion of the weld. Much success has been obtained in welding the metal in an inert shield of argon gas, thus dispensing with the flux. Brazing operations can be easily carried out using aluminium silicon rods and applying flux to the surface of the joint. The molten filler metal percolates into the joint and is held firm by capillary attraction. Soldering has so far never been carried out successfully on aluminium, other than on small areas, but interesting developments are being made in this field with high hopes of success.

Weight of Aluminium Sheets per sq. ft

1 cu. in. of aluminium weighs 0·096 lb

Imperial S.W.G.	10	12	14	16	18	20	22	24	26	28
Lb per sq. ft	1·77	1·44	1·11	0·885	0·664	0·497	0·383	0·304	0·248	0·204

Zinc

Specific gravity 7·14. Melting point 419°C.

Physical properties. Zinc is commonly known as *spelter*, not to be confused with brazing spelter, an alloy of zinc and copper. Zinc has a *bluish white colour* and a *high lustre.* The brightness of the metal is dimmed by impurities, notably iron. When pure the metal is *malleable* and can be rolled into sheets at 100–150°C. If heated to over 200°C the metal is very brittle and can be crushed into powder. This brittleness is also present to some extent at temperatures below 100°C, and for these reasons zinc should be slightly heated before rolling, folding or hollowing. It is usual to immerse zinc sheets in tanks of hot water for short periods just before these operations.

A scriber should not be used for marking out zinc sheets as it will cut the surface of the sheet and cause cracking along the scriber lines when the metal is folded. To prevent this it is usual to mark out zinc sheets with either pencil or chalk. For similar reasons folds of sharp radii should always be avoided.

A remarkable property possessed by zinc is its immunity to annealing. On heating the metal becomes softer and more workable, but when cooled it becomes brittle again. At high temperatures (above 930°C) zinc boils and turns into a vapour which burns in air with a bluish white and brilliant flame forming an oxide which floats in the air; this is known as ' philosophers wool ' and is considered harmful if inhaled in large quantities. The speed with which the metal volatilises renders welding difficult, but soft soldering can be easily carried out using hydrochloric acid as a flux.

Zinc combined with other metals forms a useful range of alloys; with copper it forms brass, and with aluminium and lead die casting alloys.

Chemical properties. Zinc is quickly oxidised by carbon dioxide and steam, but as with aluminium the oxide is corrosion-resistant and protects the metal from further attacks. Sulphuric acid dissolves zinc slowly, while the metal will dissolve readily in nitric and hydrochloric acids. With hydrochloric acid the resultant fluid is chloride of zinc, which is used as a soldering flux for many metals, and is often referred to as killed spirits.

Molten zinc has the property of absorbing iron and, therefore, pure zinc is not readily available. Most of the zinc used for metal fabrication contains 0·5–1·5% iron and other impurities. The amount of iron in the metal can be readily seen if fracture occurs. When comparatively pure the crystal faces sparkle with brilliance, but if a large amount of iron is present the crystals are less brilliant with intermingling dull areas.

Uses of zinc. Sheet zinc has a limited application in sheet metal fabrication, due to the greater suitability of other metals which have similar properties without the brittleness associated with zinc. It is, however, used almost exclusively for the manufacture of downspout heads, roof gutters, water tanks and flashings on buildings.

Metal sheets are perforated and then used in kitchen cabinets, water sieves and agricultural riddles. Zinc is used as a protective metal in both galvanised sheets and die castings, but it is not used in the food and canning industries, except where unusual circumstances prevail.

Weight of zinc sheets per square foot

1 cu. in. of zinc weighs 0.252 lb

Imperial S.W.G.	10	12	14	16	18	20	22	24	26	28
Lb per sq. ft	4·65	3·78	2·9	2·32	1·74	1·31	1·02	0·80	0·65	0·54

Tin

Specific gravity 7·28. Melting point 232°C.

Physical properties. Tin, a white metal with a faint yellowish tinge, exceedingly bright when pure, has low ductility and a tensile strength of only about 2 tons per sq. in. For this reason it is not used in its pure form for metal fabrication, but usually as a coating material for such metals as iron and to a lesser extent copper. The metal is a main constituent in the useful range of alloys of tin and lead known as soft solders which are used extensively as low temperature bonding metal between most of the commercial metals.

A strip of tin when bent emits a peculiar crackling sound known as the ' cry ', which is due to the internal friction between the crystalline particles. Solders containing a high percentage of tin also make this sound if subjected to bending. Tin is in its most malleable form at 100°C, but at temperatures above 200°C it becomes brittle and can be pulverised. At low temperatures, below 18°C, the metal is unstable. This change can be reversed by pouring hot water over the metal, or by storing in a warmed atmosphere.

Chemical properties. Tin is not affected by dry or moist air at normal temperatures, but it rapidly oxidises if heated. For this reason it is always advisable to melt the lead before inserting the tin during the process of making solder.

Hydrochloric and sulphuric acids quickly dissolve the metal and it reacts violently with nitric acid. It is also soluble in caustic soda, and combines with iron at high temperatures forming a brittle compound (FeSn) on the surface of the iron. Another brittle compound is formed when tin is in contact with copper, which in turn causes the metal below to corrode away. The corrosion is accelerated by acids at high temperatures and for this reason soldering irons should never be overheated.

Uses of tin. Sheets of tin are never used for metal fabrication; almost one half of the world's output is utilised in the production of tinplate, whilst much of the remainder goes in the manufacture of soft solder.

Tin may be rolled into a very thin foil which is ideal for preserving foodstuffs; this, however, is now being replaced by aluminium, which is a less costly material.

Lead

Specific gravity 11·36. Melting point 327°C.

Physical properties. The metal, a bluish grey colour with considerable lustre on fresh surfaces but soon dimmed on exposure to the atmosphere, is the softest metal in common use, easily cut by a knife or scratched by a nail. It is said that lead can be used for writing on paper, but it should not be confused with the lead of a pencil which is produced from a mixture of graphite and various clays.

Lead is extremely malleable, with little or no elasticity and a low tensile strength of only 0·4–0·8 tons per sq. in. Although lead is one of the heaviest metals it is so weak and soft that it can be rolled into sheets using only light rolls. A bar of steel 1 in. in diameter will hold a load of 25 tons without breaking, but a similar lead rod would break if a load of only one ton was applied. The metal has a remarkable power of elongation and would stretch considerably before breakage occurred. For these reasons lead is unsuitable for work where considerable stresses are encountered.

The metal has no affinity with iron or steel but readily combines with tin to form solder, and with antimony to form other alloys.

Chemical properties. On exposure to the atmosphere lead suffers corrosion by oxidation, but fortunately the skin so formed does not easily dissolve and thus a barrier is set up, preventing further corrosion. The metal can be dissolved in diluted nitric acid but not in undiluted acid unless it is boiling. Lead is not acted upon by hydrochloric or sulphuric acid or by hard water, but it tends to be affected by soft water after long contact. The metal is highly poisonous and should not be used in contact with food. For this reason it is dangerous to ' tin ' cooking vessels with soft solders containing a high percentage of lead.

Uses of lead. Lead used to be considered a cheap metal but in recent years it has become increasingly expensive. Water pipes were made exclusively from lead, the only disadvantage being that in cold weather when the water froze the expansion which took place was sufficient to burst the metal; today water pipes are often made of copper. Acid tanks are made from wood and then lined with lead on the parts in contact with the fluid.

Lead was also used extensively for roof covering, but the expense of undertaking such work is now hardly ever considered, galvanised iron being most frequently used in its place.

The metal can easily be welded using an oxy-coal gas flame, but the metal surfaces must be free from oxides and other impurities. It is soldered by using a filler metal containing 1 part tin—2 parts lead. This solder remains plastic for some time on heating, hence allowing the joints to be formed or ' wiped ' by the hand protected by a leather glove, and using tallow wax for fluxing.

The high malleability of lead enables it to be worked into complicated shapes with comparative ease and small tank liners are often made in one piece by beating the metal with a flat-sided wooden forming tool. No intermittent annealing is carried out, for there is little danger of the metal cracking during the working.

Lead sheets are not ordered in thicknesses but in lb per sq. ft. For instance a sheet of 4 ft × 2 ft × 10 lb lead will weigh 80 lb.

PROCESSES AND PRACTICE

The testing ground for all the theories and science associated with sheet metal work is the workshop bench. The calculations required may be considered elementary by mathematicians, and the geometrical theories of the pattern developments present no difficulties to the draughtsman. Yet, when the patterns have been drawn and the sizes calculated, it is the practical man who has the responsibility of producing the finished article and ensuring that the sizes shown on the drawing are reproduced exactly on the workpiece and are not a mere approximation.

To do this requires patience, practice and a high degree of skill, which can be developed only by years of experience coupled with natural ability. Probably the hardest and yet the most thrilling process in any trade is the production with one's hands of the ideas developed in the mind. Workshop practice assists in no small way in achieving this purpose, and its value in technical college courses should never be underrated or neglected.

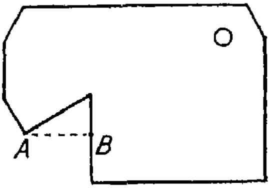
This chapter is devoted entirely to practical aspects of sheet metal work and, while it is impossible to cover them all, an endeavour has been made to give the student sufficient information to enable him to carry out his daily tasks in a more efficient way, and hence improve his own capabilities.

Marking Out

Before beginning to form metal into any desired shape it is necessary, as far as possible, to mark out the allowances for all joints, wires, flanges and the position of any bends required. Many methods of producing the lines in the correct positions are employed, and their merits are considered in the following.

(*a*) **Scribers** are made from bright hard steel rods sharpened at the end to a point, usually by grinding. In some instances scribers are made from hard brass rod and leave a faint yellow line on mild steel sheets and stainless steel. Steel scribers are not recommended for mild steel sheets unless the surface has been previously painted with a whitening solution or covered with diluted copper sulphate, owing to the difficulty of seeing the lines produced by the scriber. Scribers should never, under any circumstances, be used for marking out fold lines on zinc sheets, as the metal will have a tendency to crack when bent. Scriber lines can be more easily seen on tinplate by breathing heavily on the surface of the sheet.

(*b*) **Chalk.** Soft sticks of chalk are not often used for marking out owing to the difficulty of maintaining a sharp edge, and as the lines produced are not of an even width they give rise to errors. French chalk, however, in the form of thin rectangular slabs, sharpened at one edge to a chisel point, are used. Also, a piece of thin string covered with whitening is useful for marking off long straight lines on steel. The string is held tightly across the surface of the metal in the required position; the mid-part is raised away from the sheet, then released. When the string strikes the sheet it deposits a thin white line along the line of impact which can easily be seen.



Flat metal gauge

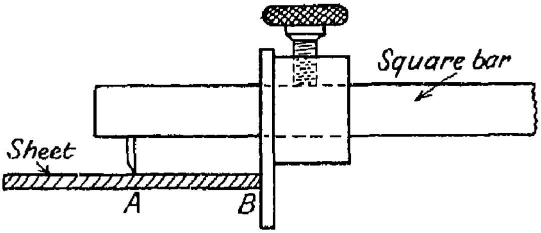


FIG. 88. Gauges.

(*c*) **Gauges.** Where possible marking out gauges should be used to ensure accurate measurement. They are obtainable in many forms. Two simple workshop gauges, easily made, are shown in Fig. 88. The first is simply a piece of scrap material cut to the shape shown. It is useful for marking off edging, folding and wiring allowances. The required measurement is the horizontal distance between the point A and the side B.

This may seem a crude method of producing a gauge, but its crudeness by no means diminishes its effectiveness. A line for folding a groove parallel to a given edge could be produced by this gauge far more accurately and more speedily than by any other method. Furthermore, the same line may be reproduced on an unlimited number of sheets and they will all be exactly the same. This would not be possible if the marking out were done using a rule and scriber, where there is some margin for error on every measurement.

Another simple marking out gauge which is very effective can be made from square section flat bar with a hardened steel needle fixed near the end. The working face is movable and can be locked at any position by means of a thumbscrew at the top. Its applications are the same as those described previously, except that the working limits are somewhat wider.

Whenever possible the time spent on marking out should be reduced to a minimum, except on single jobs. This is done by judicious setting of folding, cutting and edging machines or, if the quantity required warrants it, by making accurate templates and blanking tools to be used on presses.

Miscellaneous Tools

Bench stakes. There are many different shapes and sizes of these stakes. Selection of the correct pattern will depend upon the shape of the sheet metal article, the process to be carried out and also the size of the work. Stakes should always be kept in good condition with the surfaces free from blemishes and rust. This can only be done by frequent cleaning and polishing.

The stakes should be firmly supported by fitting them in a tapered square hole cut in the work bench, or on a special casting screwed on to it. Whatever the method of attachment, care should be taken to ensure that ample working space is available, especially between the work bench and the underneath part of the working portion of the stake. Fig. 89 shows some of the more common stakes in general use, although many others are available for special jobs. In the main the ones illustrated will be found sufficient for most types of work encountered in normal practice.

(*a*) **Pipe stakes** are useful for shaping cylinder sides and hand grooving of circular pipe joints.

(*b*) **Creasing irons** are mainly used for hand wiring of flat sheets or making false wires of thin metal. The flat surfaced end, however, is often used for closing down edges or squaring up worked joints.

(*c*) **Funnel stakes** are used for hammering and planishing tapered vessels and square to round transformers, also for hand forming of small conical shapes.

(*d*) **Anvil stakes** are for straightening wire and other material. Edges can be used for squaring up knocked up ends on boxes.

(*e*) **Side stakes** are also used for these operations. The end which is cut off at an angle may be used for flanging off edges on small blanks and also for raising up.

(*f*) **Tea kettle bottom stakes** are employed in many ways to assist in the fitting on of circular or square bottoms on liquid containers, also as underneath supports when hand wiring cylinders.

(*g*) **Tinman's mandril.** The rectangular end is used extensively for jointing box ends and also for flanging. The radiused end is employed for raising, rounding and forming large circular vessels.

(*h*) **Half moon stakes** are for flanging operations on cylindrical articles.

(*i*) **Necking stakes** are used for stretching and flanging curved surfaces.

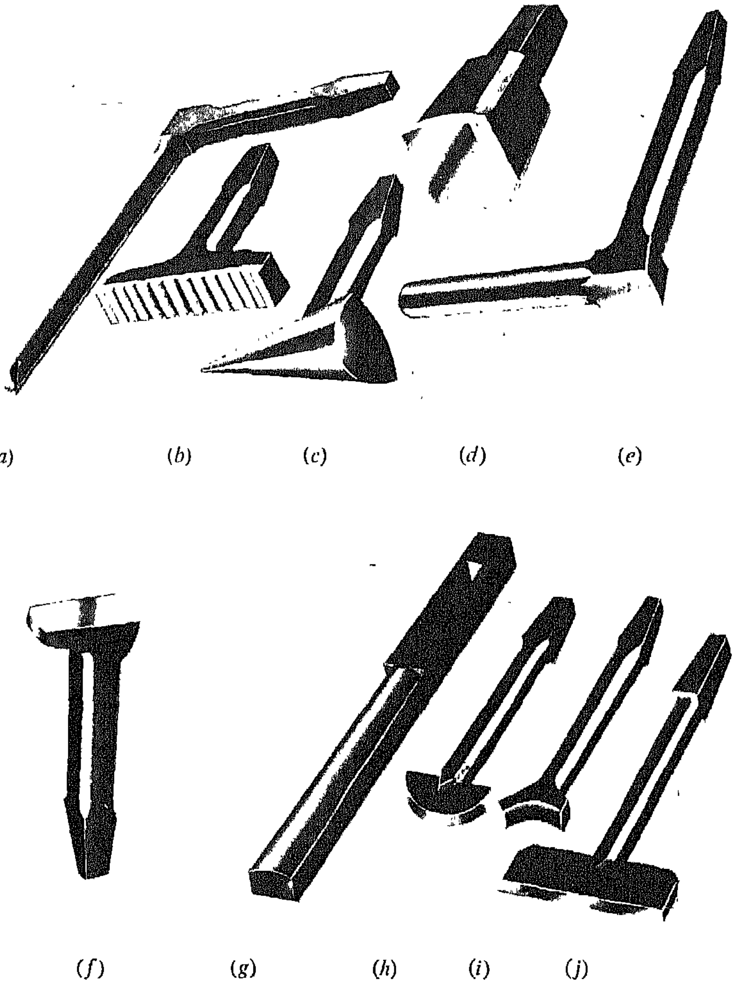


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FIG. 89. Bench Stakes.

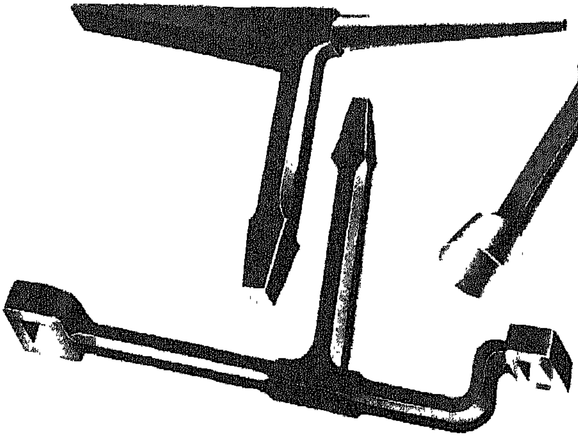

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FIG. 89. Bench Stakes.

(*j*) **Hatchet stakes** are used for creasing and folding by hand; also for flanging and the tucking in of joints may be done on this type of stake.

(*k*) **Bick irons** are extensively used for all types of operation such as wiring, knocking up, flanging, stretching and making tapered curve surfaces. They are available in many shapes and sizes to suit almost any purpose.

(*l*) A **bench horse** is a rigid support into which small irregular shaped heads are fitted to enable planishing to be carried out on double radii curved surfaces.

(*m*) **Round bottom or canister stakes.** These are suitable for working up circular shapes, round lids and levelling out flat bottoms after knocking up.

Hammers. In no other trade is the hammer employed to greater effect than in the sheet metal industry. The men who specialise in the production of components by judicious use of the hammer are often known as panel beaters and in many areas are considered to be more highly skilled than the general sheet metal worker who concentrates mainly on flat work. Metals formed by hammering are not the products of one hammer, but only a combination of many, with special ' heads ' and stakes, used in sequence; therefore, like the bench tools, hammers should be kept well polished, especially at the contact surfaces.

Before beginning to hammer any metal the surface of the material should always be clean and, if possible, annealed. Dirty material has a detrimental effect on the tools and also on the finished work. For soft metals, such as aluminium, copper and brass, the initial working is carried out with wooden or, in some cases, rubber mallets, and then finished off with the steel hammer. Fig. 90

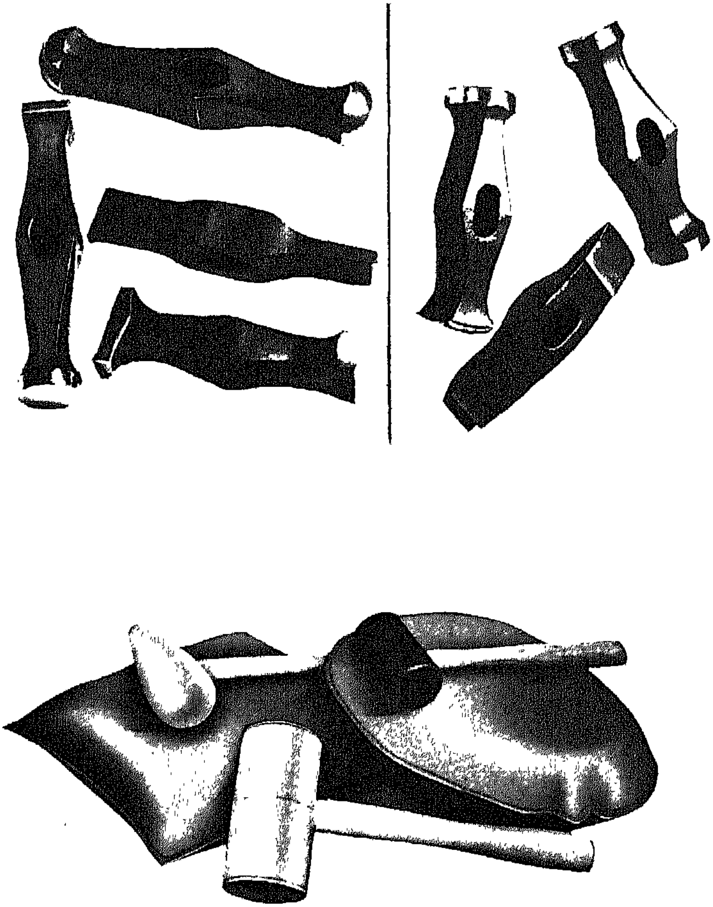

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Fig. 90. Hammer Heads and Mallets.

illustrates some of the hammers in general use, their applications being as follows:

Type of Hammer	*Uses*
Smoothing	Levelling and working over joints
Stretching	Stretching of metal
Creasing	Closing down joint edges
Hollowing	Working up sharp radii
Planishing	Smoothing off finished work
Riveting	Forming rivet heads
Cross paning	Thinning down material
Blocking	Heavy hollowing of thicker meta

Mallets can be obtained in many forms: plain with parallel heads and round, flat faces or with one face machined to a hemispherical form. Pear-shaped mallets are used in connection with sand bags covered with buffalo hide for metal shaping; they are fitted with hickory shafts and the curved working faces are formed in many different radii.

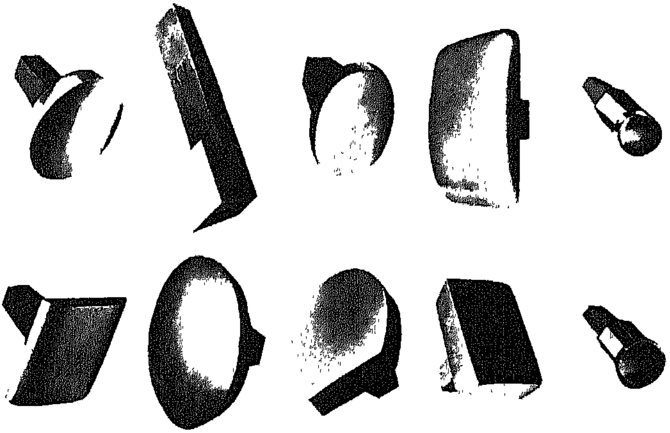

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Fig. 91. Panel-beating Heads.

Mallets are available from 1½ in. to 4½ in. in diameter, rising in steps of ½ in., although larger diameters can be purchased for special two hand working.

Other small tools. A selection of these is given in Fig. 91. Each one has its own special use and is an essential part of the sheet metal worker's equipment.

Cutting Tools

Whatever form a material may take, it is important to use the correct methods of cutting to ensure that maximum efficiency is obtained. Hand snips and shears are the most common tools used and of the following types each has its own field of application.

(*a*) **Straight Snips.** These are used for straight cuts and for trimming surplus metal from articles where the shape will allow access and free manipulation of the handles. They are also used extensively for notching or cutting out joint allowances at fold lines and overlaps. The size of the snips is proportional to the thickness of metal to be cut. The heavier the gauge the greater will be the length of the handle, to enable the metal to be cut with a minimum of personal effort. Another important factor in determining the size of snips required is the maximum shearing stress that the metal will stand, for a much longer pair will be required to cut, say, a piece of stainless steel than an aluminium sheet of similar thickness.

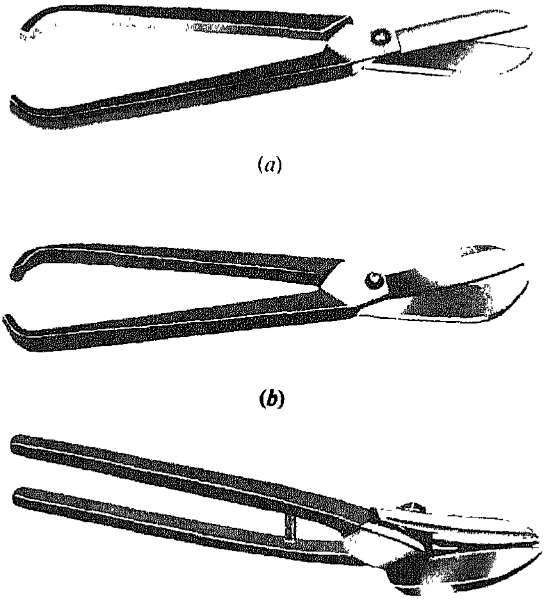

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FIG. 92. Snips.

Snips are made from high carbon steel or stainless steel and have ground and polished blades. They are available in sizes from 8 in. to 14 in. in steps of 1 in., but larger sizes are produced to meet special requirements.

(*b*) **Bent Snips** are, as the name suggests, curved on the cutting blades and are used for cutting circles or irregular shapes including internal diameters. They have only a limited use, but are nevertheless an important piece of the tool box equipment.

(*c*) **Universal** or **Gilbow Shears** are a combination of the two types previously described and are of robust design. The handles are offset, thus allowing cuts to be made in any direction with a minimum effort. The sheet is kept flat and the operator's hand is always clear of the metal, the cut being followed from above. Gilbow snips may be right or left handed, this being defined by the position of the upper cutting blade in relation to the lower one.

Hints on metal cutting by hand shears. The handles should be gripped as far as possible from the blades, although care should be taken to keep the inside part of the hand clear of the turned-in part of the handle when the cut is made, otherwise the flesh may be pinched and bruised. To prevent this possibility, two fingers should be free and take up position between the handles. On terminating each cut, the free fingers may be used for relieving the pressure and forcing the handles open again, hence allowing them to take up position for the next cut.

To produce a continuous edge free from burrs, always hold the shears at right angles to the workpiece and endeavour to maintain the lower blade in a position square to the surface of the metal; it is the upper blade which contributes most in the shearing. The shears must never close completely in any one cut, otherwise the metal will become distorted along the edge.

Shear cutting is purely a one-handed operation and the free hand should be used to hold the work and guide it between the blades. For thick materials it is common practice to grip the lower handle in the vice to enable a greater thrust to be applied to the upper arm. This is not recommended, as the shears quickly become strained. It will be found safer to use a larger pair or carry out the cutting by a mechanically operated tool.

Bench shears. A quicker and safer method of cutting thick metal is by using hand-operated bench shears. These have a fixed lower blade held into a casting by countersunk screws. The upper blade is movable, working on a pivot and attached to a handle. The full length of the blade should never be used where the metal warrants several cuts. Otherwise, as in the case of hand snips, a rough finish will be produced.

These types of shears vary in shape to allow curves to be cut with ease, and also the bottom blade is drilled and used for wire cutting.

Another type of cutting tool, known as electrical vibrator shears, is used for complex curves. The machine consists of a small electric motor enclosed in the machine body, which vibrates a pivoted blade by means of a cam and spring attachment. The rate of oscillation is relatively high and the pivoted blade works in conjunction with a fixed one, both being easily removable for sharpening. On reasonably thin material this machine is ideal for cutting flat sheets in practically all types of curves. The metal is held in a fixed position whilst the machine is manipulated to enable the blades to follow the required line of cut. One drawback in using this type of machine for long periods is that it must be carried in the hand; this becomes very tiring.

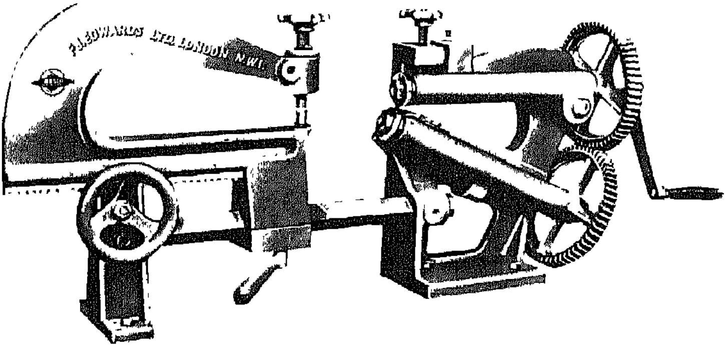

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FIG. 93. Circle Cutting Machine.

Circle cutting machines are used, as the name suggests, for producing circular blanks from sheet metal. The one illustrated in Fig. 93 is a simple hand-operated type used mainly on thin material. It consists of two heavy cast iron frames, one carrying the cutting wheels driven by the handle revolving the bevel gears, and the other carrying the clamp, which may be adjusted by means of a thumbscrew to suit the diameter being cut. On other types of machine the clamp is replaced with a pallet to prevent the centre of the produced circle being marked.

When in operation the blank is first inserted between the clamp or pallet in such a way that it is free to rotate, and adjusted on the sliding frame to give the correct diameter. The wheels are then lowered by means of the top screw until the blades just shear through the plate and then they are rotated by the handle until the blank is cut from the stock sheet. Cutters used on this type of machine are hardened and made from good quality cast steel, and circles from 2½ in. to 14 in. may be cut with ease on metal up to 24 S.W.G. on a small machine weighing only 40 lb. Like most other sheet metal working machine tools, circle cutters are available with power drives when quantity production is required.

Guillotines. Guillotines vary in shape and size from the small hand- or foot-operated types for cutting light gauge tinplate to the heavy duty power-operated types used for shearing boiler plates, and are used extensively for straight line cutting. The principle on which all the machines operate is the same; an upper blade which is offset slightly from the horizontal plane descends vertically towards a fixed bottom blade which is fitted to the bed of the machine, thus shearing off the metal along a line where the two blades pass.

Most machines are power driven by small motors which can be operated by a foot pedal, which, when depressed, causes the blade to move up and down until the foot is removed. Adjustable stops are fitted at the back of the machine to allow work of a predetermined width to be cut. These stops may be adjusted in a variety of ways according to the design. On small machines, however, the stop consists of a flat bar bolted at the ends to slotted cast iron supports. They are adjusted by measuring off the required distance from the cutting edge of the bottom blade to the face of the stop, and then tightening up the bolts. At the sides of the cutting bed are guides or gauges set square to the guillotine blades. These are to maintain the cut edge square with the sides of the sheet. When only a few pieces are required it is good practice to dispense with the back stops and work to a line scribed on the work. But for quantity production stops have proved to be an invaluable aid to cutting.

All guillotines are fitted with protection guards to prevent accidents. These should never be removed under any circumstances, or the operator will expose himself to the risk of serious injury or permanent disablement. Care also should be taken to see that all small tools such as rules, scribers and calipers are not left on the machine when it is in operation, for once the blade descends it will cut anything within its path.

Clearance. This is the distance between the cutting edges of the top and bottom blades and should be adjusted to suit the thickness of metal being cut, using engineers' feeler gauges. On many machines the clearance is determined by the makers and is not adjustable, hence the machine is only suitable for working between certain thicknesses of plates. For thin sheet metal no clearance is allowed, but for thicker work the table below gives the clearance required.

Metal thickness	*Clearance between blades*	
	Non-ferrous metals	*Steel*
in.	in.	in.
0·035	0·0015	0·0018
0·065	0·002	0·0025
0·1	0 0022	0·003
0·125	0·003	0·004
0·25	0·0035	0·007

The clearance will have a direct effect on the condition of the cut edge of the material. When correct this will be cleanly cut and have no sharp edges. If, however, too much clearance is left between the blades, a sharp ragged burr will form on the lower edge of the sheet, or instead of cutting the metal the blades will simply turn it over at right angles along the line of action. Insufficient clearance will create excessive pressure at the upper surface of the metal leaving a bevelled edge and a rounded corner.

Joggling, Swaging, Jennying and Wiring

Joggling (Fig. 94(a)) is carried out to allow two plates to lie in the same plane when jointed together. It can be adapted for flat or cylindrical work. In the case of flat sheets joggling is done on the folding machine by forming two slight bends along the edge of the metal; it is used extensively for riveted or soldered lap joints and for flush doors on small cabinets. Should the metal need to have a wired or turned over edge, then the depth of the joggle must be increased to suit the requirements (Fig. 94(b)).

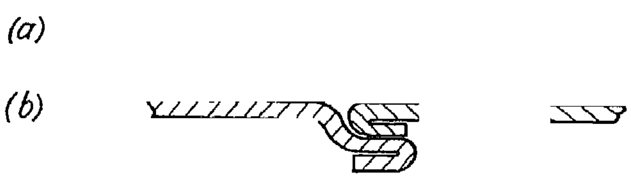

FIG. 94

Joggling on cylindrical work to allow the faces of the metal to lie in the same plane can be effectively carried out on a jennying machine. It may be external or internal (Fig. 95). The joint should be sealed by riveting or soldering, but joggling is never used for brazed joints where the overlap must be at least equal to the pipe diameter.

Swaging is done to raise a bead or moulding on sheet metal and hence stiffen it. The process is mainly used on tanks and circular vessels to give additional strength to the sides of the vessel, although motor body parts and flat sheets are sometimes so treated. Nowadays, swaging is purely a mechanical operation, carried out on hand-operated or power driven machines fitted with changeable swaging wheels of different shapes. The wheels are made in pairs, inner and outer, fitted to the revolving shafts of the machine. To operate the machine the metal is placed between the wheels, which can be closed, until a slight depression is made on the surface. It is then passed through the wheels and the required shape of the moulding is formed.

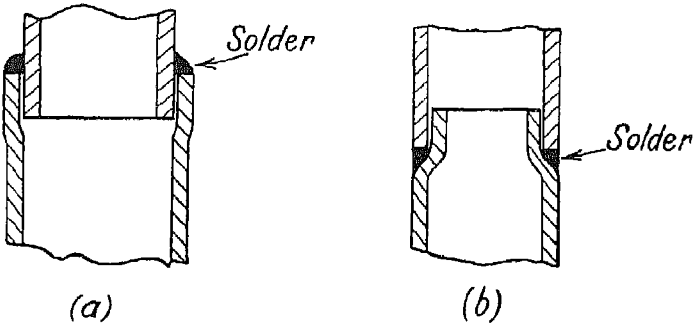

Fig. 95

The swaging machine is designed with two driving shafts mounted on long bearings which are geared at the main frame end. A hand screw brings the gears into mesh. In order to keep the work parallel to the swaging wheels, a movable guide is held in position between the shafts by cast iron supports. For thin material (up to 18 S.W.G.) the force required can be easily supplied by hand, but for thicker work power is used to drive the machine. In many workshops the swaging machine is often referred to as a ' beading ' machine.

Jennying is a method of turning up small edges on circular blanks or cylinders, and it is carried out on a machine known as a spinning jenny. This machine is probably the most used bench tool in all sheet metal working operations and it can be adapted in many ways to suit special circumstances. The spinning jenny is of the same design as the swaging machine, but only hand-operated types are available. Some degree of skill is required in its operation.

The jenny rolls or wheels are made in pairs; the upper wheel is in the form of a thin cylindrical blade, whilst the lower wheel has a small recess cut into the end all round its circumference. A hand wheel at the top of the machine is used to apply the pressure on the metal when placed between the rolls. In order to keep the edge to be spun up parallel, a small adjustable gauge is fitted, and this is set at the required distance from the top wheel. The metal is held between the wheels and slight pressure applied. When this has been done the wheels are revolved by turning the handle with one hand and holding the metal firmly against the gauge with the other. If the correct pressure has been applied, the surface of the sheet should be marked, but not cut, by the wheels and still held tightly between them. The handle is then turned at an even speed whilst the metal is forced slowly upwards and over until the edge reaches the desired angle of deflection. It should be remembered that the pressure required for throwing off the edge is created simply by forcing the revolving metal upwards and over and not by means of the hand wheel at the top of the machine. Should too much force be applied by the latter the metal will be cut instead of formed.

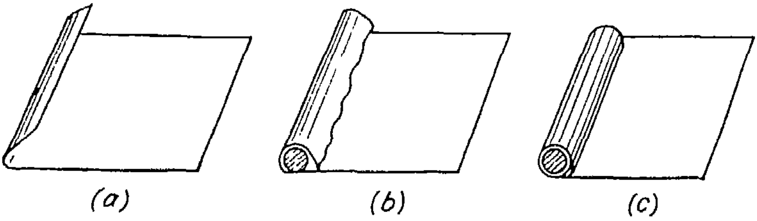
(a) (b) (c)

FIG. 96

Wiring is the process of forming a sheet metal fold round thin wire to give extra strength to the edges of an article and also to improve its appearance. It may be carried out by hand or by machine methods, or by using a combination of both. The allowance of material for wiring is 2½ times the diameter of the wire or twice the diameter of the wire plus four times the thickness of the material. In practice the authors successfully use the former for flat sheet work and the latter for cylindrical work where the edge is stretched over by hand. This distance is marked off on the sheet and the metal folded as shown in Fig. 96(a). The wire is placed in position and the metal is beaten over by a mallet as in Fig. 96(b). Finally, the edges are closed by passing the metal through a wiring machine, which is similar to the one for swaging and jennying. Should a hidden joint be preferred, the metal is turned over and repassed through the machine. A universal machine which can be used for all the processes, swaging, jennying, wiring and closing, has been developed and is complete with 12 pairs of wheels which are interchangeable. A diagram and full particulars are given in Fig. 97.

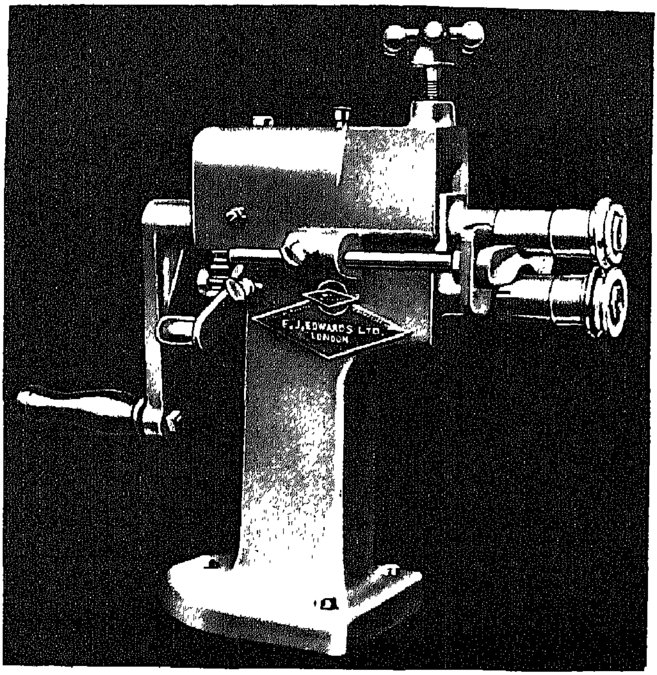

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FIG. 97(a). 'Besco' Universal Swaging, Wiring, Jennying and Closing Machine.

The machine should be bolted firmly to the bench and both spindles well lubricated. When fitting wheels, note that the lower spindle has a left-hand thread. The setting of the lower spindle to give correct alignment to the various types of wheels is by means of the lever at the back of the machine. See that the clearance between the male and female wheels is evenly distributed. After sliding the lower spindle into the correct position, secure firmly by the screw at the side of the main frame. This screw varies in type according to the size of machine.

The gauges consist of two strips, the deeper of the two for use with cylinders, and the narrow strip which fits into the front of the former for use when wiring or jennying, as it can be set right up to the inside face of the wheels. On the larger size machines, the first part of this gauge is of the deep apron type to handle large cylinders. From size 4 upwards, there are two positions for the brackets holding the gauge, so that full use may be made of the deeper throat. For normal work, the brackets should be in the front position as they act as tie rods. When using the circular guide attachment for wiring or jennying the edge of discs, see that the rod between the guides is level with the lower clamping pallet in the bow and the wheels of the machine. Set the centre of the disc in the clamping pallets in the bow, and slide the bow towards the wheels until the edge of the disc is between the wheels to give the required turn-up. Lock the bow in position by the two thumbscrews. As the wheels are rotated and pressure is applied to the top spindle, the disc will spin between the wheels. The operator then tilts the bow, thus spinning the edge on the disc to the required angle.

Twelve pairs of wheels for various operations are supplied with each machine, the profiles being shown in Fig. 97(b). If the operator is not skilled, it cannot be stressed too much how important it is to practise on old pieces of metal with the various types of wheels.

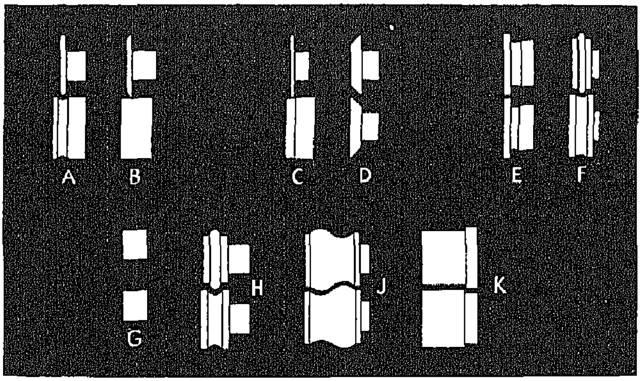

FIG. 97(b)

A. Wiring rolls for forming U-seatings for wire.
B. Closing wheels for closing metal round wire.
C. Jenny wheels.
D. Paning down wheels.
E. Bottom closing wheels, 1st and 2nd operations. Bottom wheel of B is used for both operations.
F. Swage wheels.
G Collars or distance pieces for use with narrow type wheels.
H. Swage wheels.
J. Swage wheels, with ogee bead.
K. Necking wheels. Can also be used for joggling.

In the earlier years of this century wiring was essentially a hand working process. The edge of the metal was flanged over, using a hand hammer or mallet, then the wire was inserted and the final folding took place by beating the metal round the wire on a bench tool known as a creasing iron. The face of the creasing iron was recessed at regular intervals with grooves of varying diameter to hold all sizes of wire.

Today, however, this method has been replaced to a great extent by power driven or hand-operated machine tools. Although in many cases isolated jobs are still produced by the former methods.

In modern workshops machines have been designed to carry out the whole operation. The metal and wire are fed in at one side, and the machine, which consists of a series of rollers, folds the metal, inserts the wire and finally secures it at great speed with a high degree of accuracy.

Generally this method is used almost exclusively by large firms on mass production, but smaller workshops rely mainly on the skill of their individual craftsmen to carry out the wiring operation.

For wiring of straight sides the metal is first edged up at right angles in the bench folder, and the wire secured by tapping the metal over lightly with a hand mallet.

It is then passed through the spinning jenny to smooth out the creases and tuck the edge under. If the edge of the metal is required to be ‘ hidden ’ (on the inside of the work), the plate is turned over and repassed through the jenny (Fig. 98).

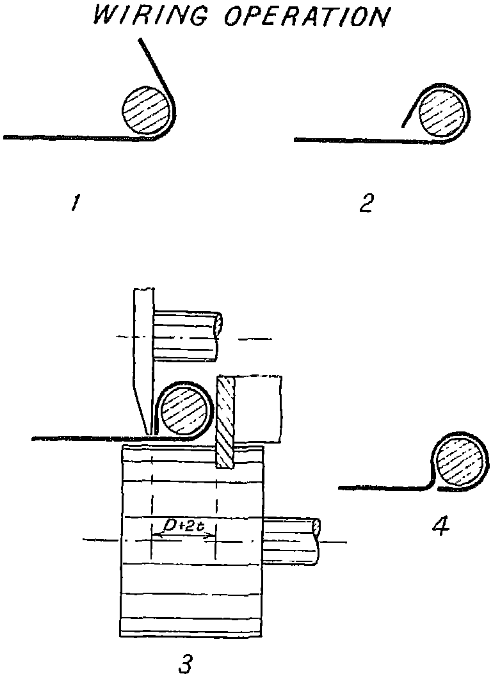

FIG. 98

For this method to be used the bottom grooved roller of the machine must be removed and a plain cylindrical roller substituted. The distance required between the edge of the top roller and guide plate being equal to the wire diameter plus twice the thickness of the metal.

Folding allowance for wired edges. The initial allowance of 2½ times the diameter of the wire will be found accurate for work produced by the above method, and if added to the required length of the sheet no appreciable error will be found on the finished work. For circular work or curved edges the first operation of turning up the edge cannot be done in the folder, hence hand working or judicious use of the spinning jenny is resorted to. When this occurs an allowance of twice the diameter of the wire plus 4 times the metal thickness may be used instead of the former method, although experience will show that good results will be produced by either method.

Wiring before rolling. Whenever possible wiring should be completed while the metal is flat, before it is rolled into a cylinder. Furthermore, it is bad practice to allow the ends of the wire to terminate at a position near a joint. To prevent this the wire is left protruding somewhat at one side and similarly shorter at the other, so that when bent the wired ends interlock in the same way as toy railway lines.

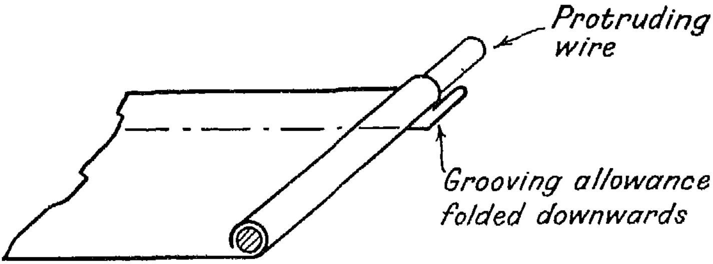

FIG. 99

Where wiring is carried out for a grooved edge joint, particular notice should be taken of the type of bending rollers used. Some are designed to turn the metal over the top roller and others underneath. The method of recognising which way the rollers bend the metal is that when the top front roller is grooved for wiring the metal turns underneath and encircles the lower front roller. Similarly when the grooves are on the bottom roller the metal turns upwards and over the top one of the front pair. This means that a definite manner of edging the material for the grooved joint must be employed; also the wire which is left protruding must be at the end where the edge of the metal is turned *down*, for grooving the seam (Fig. 99).

If the metal is rolled without any precautions the small portion between the edge of the wire on the short side and the end of the plate tends to become crushed or split by the pressure of the rolling. A practical tip is to leave the wire longer at both ends, and nick saw it at the place where it is to be removed. When rolling has been carried out twist the end of the wire with a pair of pliers until it breaks inside the metal surround. It can now easily be withdrawn, leaving a perfect hole in which to insert the protruding end. All wire used should be clean and have a slightly greasy film on its surface, so that it can be pushed into position with minimum effort.

Box Wiring

When a square or rectangular box with an open wired top is to be made in one piece, it is advisable to fold the metal as shown in Fig. 98(1) before bending the sides of the box at right angles. The final operations are carried out by hand when the corner joints have been completed. In the same way a wired lid may be considered to be a shallow box and the same procedure adopted. However, if a strong durable job is required, it is usual to make the box in sections, the bottom and two longest sides being produced in one piece, whilst the ends are secured by means of knocked-up joints; the joints act as stiffeners at the corners and across the bottom. Wiring of a box of this design may be completed whilst the sheets are flat, so it is a simple process. The body, which consists of the bottom, two of the sides of the box, and the two ends, is cut to size and the edges wired, care being taken to ensure that the wire is perfectly straight and clean; the wire on the edges of the body is withdrawn after the metal has been folded for the joints and bottom edges.

It is important that the wire passing through the end pieces should have a length equal to that of one side plus the width of the end of the box, and should protrude by the same amount at both ends of the metal. The protruding parts are then bent at right angles to the end pieces and fitted into the holes left on the body where the other wires have been withdrawn. Finally the knocked-up joints are completed.

Backlapping

When the lid of a trunk or box is required to overlap the sides of the body, then the wiring must not be at the top edge but at a distance down the sides, thus enabling the top to be fitted neatly over the end of the box and give a pleasing finished appearance.

The method of wiring for these conditions is known as **backlapping** and has many applications in sheet metal work. Wire of a smaller diameter than for the body is used for the lid, to ensure a perfect fit. Fig. 100(a) illustrates a cross section of such a box with the lid fitted into position, where it will be seen that an allowance of the width of the backlap plus the wiring allowance must be added to the required finished depth of the box.

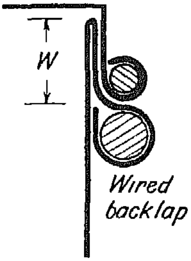

FIG. 100(a)

To complete a backlapped wired top the operations are carried out as follows (Fig. 100(b)):

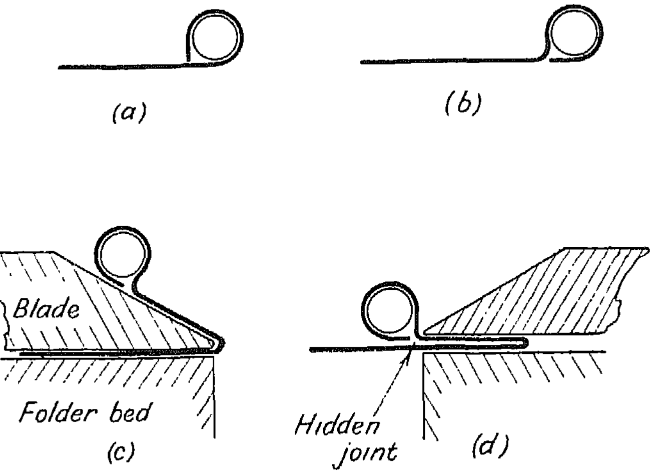

FIG. 100(b)

(*a*) Wire edge in usual manner.
(*b*) Turn plate over and form crease parallel to wire, using jenny with plain cylindrical wheel.
(*c*) Fold over top edge as far as possible in folder.
(*d*) Completely flatten edge.

An excercise which incorporates all the above processes is illustrated in Fig. 101.

Bending and Folding

The difference between bending and folding is so slight that we can assume that both are carried out with the same purpose in

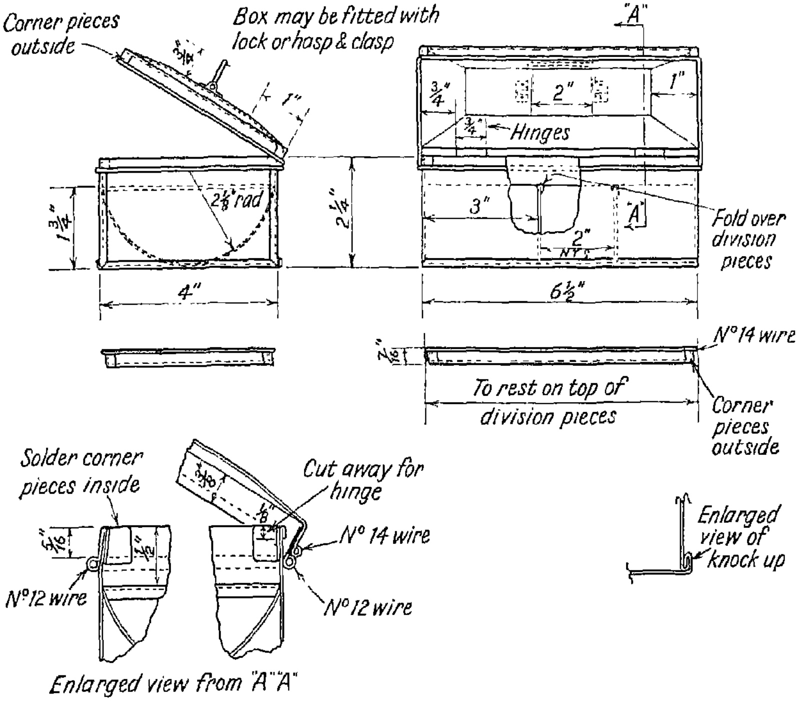

FIG. 101

view, i.e., to deflect a metal from one flat plane to another and to keep it there permanently. If the deflection is sharp, say along a given line, and the radius is small, we say that the metal has been folded. Should however the curvature be large and the deflection cover a large area, we term it as bending. A deflection through any angle of a hollow body such as a pipe is also termed bending.

There are many methods of folding and bending metals, probably the most simple being to fasten one part in a vice and force the other into position; this is often done with thin sheet metal strips. An extension of this is the bending bar, which consists of two parallel stud bars held together by bolts near the ends. The metal to be folded is placed between the bars and the nuts tightened down; the metal can be bent along the predetermined line. Additional support can be given by gripping the bars in a bench vice.

Folding and bending machines vary considerably in size from the small hand type, capable of bending up to 16 S.W.G., to the power driven type used for heavy duty. They are usually fitted with stops to enable the desired angle of fold to be reproduced on repetition work. The position of the bend is determined by setting a flat parallel bar on a cast iron frame at the back, so that once set up the machine may be utilised to reproduce any quantity of similar folds all at the same angle and in the same position on the sheets. Such a machine is shown in Fig. 102(a). Working on the same principle is a special type of machine known as a 'finger folder'. The folding blade is made of small adjustable sections which are removable, and any size of box can be formed within the capacity of the machine, including the complete right-angled bend of the fourth side. This is not possible with an ordinary type of

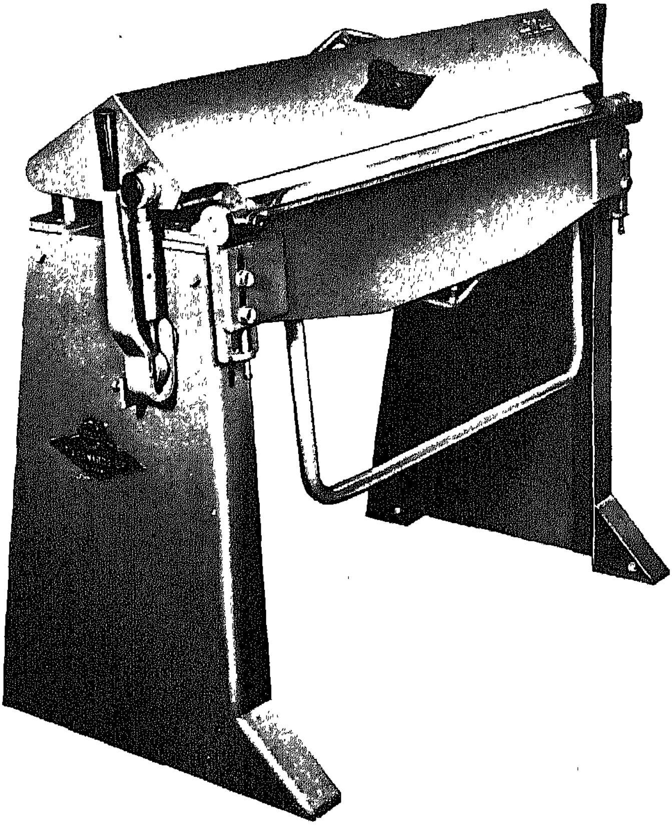

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FIG. 102(a). Bending and Folding Machine.

folding machine due to the metal on the sides previously bent fouling the blade.

For simple right-angle bends on light sheet metal a small machine is used which can be fastened on the work bench by means of screws (Fig. 102(b)). This is known as an angle bending machine and is operated by placing the metal between the blades and lowering the top by means of a hand lever. The folds may be either sharp or of slow radii according to the type of blades used, but like the ordinary folding machine some difficulty is encountered when the complete folding of a box is required.

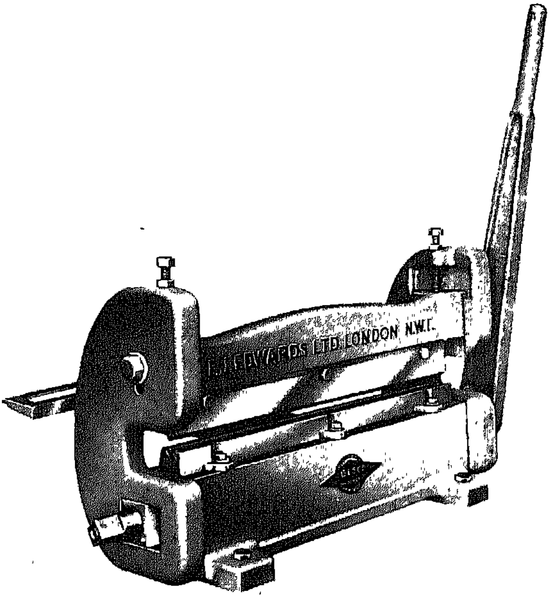

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FIG. 102(b). Angle Bending Machine.

Another type of bench folder used extensively in edge preparation for grooved joints is illustrated in Fig. 102(c), and consists of a cast iron frame supporting a steel cylindrical roller. Pivoting on the frame are the folding blades, which may be adjusted by a thumbscrew to give any desired lap. When not in use the blade holder lies horizontally on the bench, thus allowing the metal to be fitted into position between them. On raising the holder by means of the handle the metal becomes trapped between the roller and the blade, thus forcing the edge over. Edges may be folded to include the

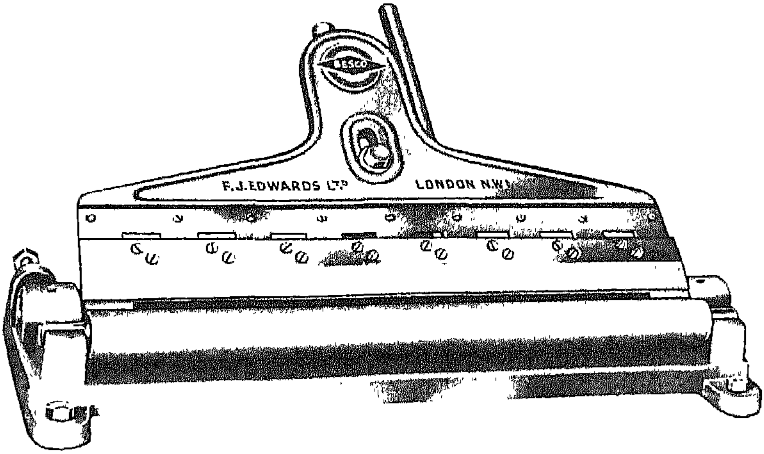

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FIG. 102(c). Roll Type Folding Machine.

180° required for grooving or if required the angle may be reduced; this however is not usual for there is no way of estimating any angle less than 180° other than by guesswork. This does not in anyway reduce the usefulness of the machine, for in most sheet metal workshops the bench folder is used exclusively for the edge folding of grooved joints.

Bending rollers are used in the fabrication of curved sheet metal panels and complete cylindrical bodies. A machine consists of three rollers; the front two are in vertical alignment and the top one is adjustable to grip the material. The back roller is also adjustable to form curves of different radii.

Some machines are made with swivel bearings on either the top or bottom rollers to allow complete cylinders to be rolled and then removed without altering the shape of the metal. Bending must be above the rear roller if the top front one is removable and below the rear one if the bottom front roller is removable. The top and back rollers are provided with five grooves of various radii to allow material which has been previously wired or folded to be rolled without damaging the metal or marking the rollers.

For general sheet metal work all machines are hand-operated with the two front rollers driven by steel pinions.

The gears are keyed to the shafts and are often enclosed by sheet metal guards, whilst the rear roller rotates freely in brass bearings.

The Rolling of Tinplate

When a curved surface is to be produced on tinplate the metal must be first submitted to a process known as ' breaking-in '. This consists of passing the metal several times through the bending rolls, and changing the direction of the curvature with each pass. The radius of the curvature should only be slow, but the more often the rolling the better the surface of the finished article will be.

Tinplate is produced by depositing a thin film of tin on the surface of a soft iron sheet; when bent the tin tends to develop in small flat parallel sections while the iron below forms a slow curvature. This gives the appearance of small cracks running across the surface of the sheet, and once they appear they are difficult to remove. By continual rolling the tin surface-metal becomes more ductile and able to follow the required curvature without showing the rolling marks, and for this reason the breaking-in process of the metal should never be overlooked or in any way hurried if good workmanship is to be retained.

Folded Metal Joints

These have an extensive application in the joining together of thin sheet metal components, but on heavy gauge material their use is limited and is replaced, to some extent, by other methods of jointing such as riveting, brazing or welding. The process of making folded joints falls into three main categories:

(*a*) Folding the metal along the edges to be joined.
(*b*) Interlocking the component parts.
(*c*) Securing the joint in such a way that it cannot be easily pulled apart, and also ensuring that the metal surfaces are in contact all along the joint.

Joints of this nature may be made by hand working of the metal, using hand-operated tools or in the case of mass production by automatic jointing machines. The latter process, however, does not now fall into the category of practical sheet metal work and therefore will not be discussed.

There are several types of folded joints in common use, each having its own particular application, although a completed workpiece may consist of a combination of several of these joints. Probably the most used is the *grooved joint* which enables flat sheets or the edges of circular or flat sided containers to be secured. In the forming of this joint the metal is folded as shown in Fig. 103(a), care being taken to ensure that the edges lie horizontal to the sheet. The bent over parts are then interlocked and formed into a joint, by forcing the metal into the shape shown in Fig. 103(b). The forming is carried out by a tool known as a *groover* or a hand-operated machine of the same name.

A grooving tool is made from cast steel and has a rectangular working face. This has a recess of the same shape as the joint machined into it, the width of the recess denoting the size of the tool. Grooves are supplied in sizes ranging from $\frac{1}{8}$ in. to $\frac{3}{4}$ in. in steps of $\frac{1}{16}$ in., but larger sizes are obtainable for special work. The formation of a grooved joint presents little difficulty if thought is given to the initial folding of the edges. For instance, if we fold over a width W and form the joint shown, it will be clearly seen that the final completed width will be somewhat greater than W, i.e., the

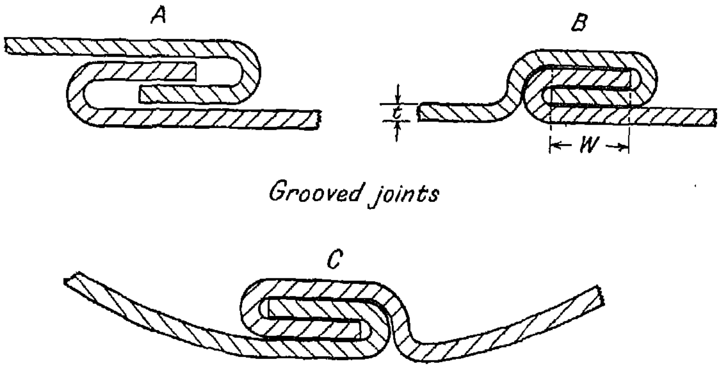


FIG. 103

final width of the groove will have a minimum value of $W + 3t$, where t is the metal thickness. This allowance for the thickness of metal must be taken into account when selecting the grooving tool to fasten the joint. It follows that if a joint is required to have a finished width of, say, $\frac{1}{4}$ in. on 24 S.W.G. material, that the initial fold line must be $\frac{1}{4}$ in.—$1\frac{1}{2}$ times the thickness of the metal from each edge of the sheet

$$\begin{aligned} 24 \text{ S.W.G.} &= 0{\cdot}022 \text{ in.} \\ \therefore \text{ Allowance for folding} &= 0{\cdot}25 - 1{\cdot}5 \times 0{\cdot}022 \text{ in.} \\ &= 0{\cdot}25 - 0{\cdot}033 \text{ in.} \\ &= 0{\cdot}217 \text{ in. from edge.} \end{aligned}$$

Similarly, if the metal had been folded over 0·25 in. from the edge, then the width of the **finished joint** would be

$$0{\cdot}25 + 0{\cdot}066 = 0{\cdot}316 \text{ in. minimum.}$$

Fig. 103(b) illustrating the closed joint also shows that the edges of the material do not fit accurately into the folding curve, and this further affects the tolerance between the groover size and the marking off allowance in the initial stages of making the joint. Hence a further clearance is essential between the tool and the metal. As previously stated, grooving tools are only supplied in steps of $\frac{1}{16}$ in. and practical experience has shown that it is necessary to use a tool $\frac{1}{16}$ in. larger in size than the folding allowance on the metal.

On light gauge material of 26 S.W.G. and above, a thin strip of scrap material of the same gauge is placed between the groover and the joint. The action is twofold.

(1) It ensures a tight fitting joint.
(2) The metal surface is protected against any cutting or abrasion due to sharp edges on the tool.

A completed grooved joint has a total thickness of four times that of the metal. Sometimes a particular job will warrant the joint being on the inside face. To enable this to be done a round bar, slotted along its length is substituted for the grooving tool. The metal is then folded and interlocked, placed on to the bar and hammered into the slot. In this way the joint is formed inside the workpiece as at C and cannot be seen. On flat sheets it will be found that, whilst the joint cannot be pulled apart, there is some lateral movement. To prevent this, the metal on the top of the groove should be spotted at intervals with a centre punch.

Cylindrical sheet metal shapes are often joined to circular bases by means of folded joints, a typical example being that shown at A, Fig. 104. This is known as an *edged-on joint* and must be finally secured by soldering or brazing. Reference to the diagram will show that the circular bottom plate must be at least 2 thicknesses of metal plus 2 widths of the cap larger in diameter than the outside diameter of the cylinder. Hence the total diameter of the blank must be

$$D + 2t + 2C$$

where D is the inside diameter of the cylinder.

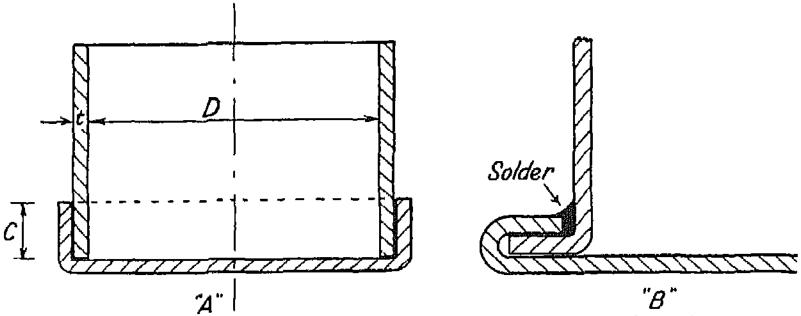
t is the metal thickness.

C is the depth of the cap.

For a joint on a 10 in. inside diameter cylinder 20 S.W.G. and $\frac{1}{4}$ in. cap, the blank size will be:

$$10 + 2 \times \cdot 036 + 2 \times \cdot 25$$
$$= 10 + \cdot 072 + \cdot 5$$
$$= 10\cdot 572 \text{ in. diameter.}$$

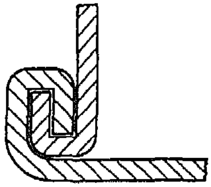
The edged-on joint is comparatively weak and where extra strength is required the *peened-down joint* shown at B may be used. In this case the cylinder has an edge thrown off and the bottom worked over as shown. The circular blank for the bottom must have a diameter of $D + 2t + 4C$, and this is jennyed up at right angles at a distance equal to C, as in the previous example. It is then fitted to the bottom of the cylinder and peened over. This may be done by a peening or bottoming machine, or by hand using a special hammer known as a paning hammer. This type of joint usually



Edged on bottom

Peened down bottom

Joints suitable for cylindrical articles



Knocked up bottom

FIG. 104

requires soldering if watertight components are desired. It is also employed for connecting sheet metal ducts and pipes, and for securing the joints of lobster back elbows.

A third type of bottom joint, known as a *knocked-up joint*, is also shown in Fig. 104. This is the strongest of the three illustrated and is constructed in the same manner as the peened-down joint. Then the whole of the metal edges are turned over and lie parallel to the sides of the cylindrical body.

Miscellaneous Joints

Fig. 105 shows other joints used by sheet metal workers. The allowances for determining the initial sizes of the plates are calculated in the same manner as those described for the edged-on and peened-down joints. Each has its own special field of application, and the decision on which type of joint to use on any particular workpiece will depend upon the following factors:

(*a*) The metal thickness.
(*b*) The quality of finish on the finished work.
(*c*) The ultimate strength of the joint required.
(*d*) The size of the work.
(*e*) The selling price of the product.
(*f*) The machinery and tools available in the workshop.

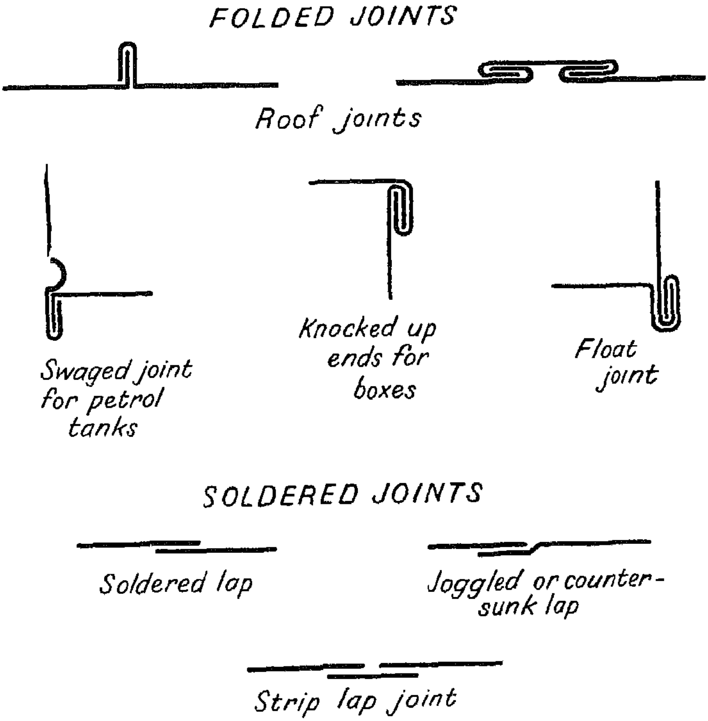

FOLDED JOINTS
Roof joints
Swaged joint for petrol tanks
Knocked up ends for boxes
Float joint
SOLDERED JOINTS
Soldered lap
Joggled or counter-sunk lap
Strip lap joint

FIG. 105

From this list it will be seen that the arrangement of the joints on any particular job may prove important when it comes to selling the articles at competitive prices. Hence the ideal would be to design the work so that it is capable of serving its purpose with high efficiency and may also be sold at a reasonable price.

Notching

Where worked up joints overlap each other or meet wired edges, some metal near the overlap must be removed to ensure a flush finish. The practice of removing this surplus material is known as notching and upon its accuracy will depend the final appearance of the finished workpiece.

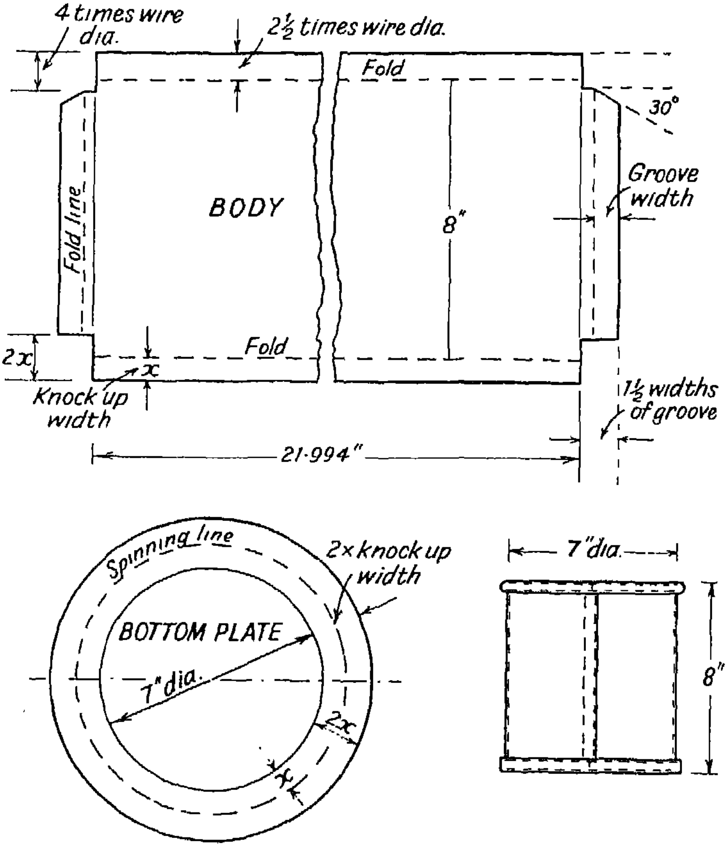


Fig. 106

For a typical example let us consider the making of the templates for a 7 in. dia. cylinder of 8 in. completed depth, with grooved vertical seams, knocked up bottom and wired top. Referring to Fig. 106, first draw the essential material to give the required size, then add the joint and wiring allowances, and finally carry out the notching to enable the work to be produced in such a way that the parts will fit accurately together.

This may be done as follows:

Body

(*a*) The length of sheet required is 7 × π in.

$$= 7 \times 3{\cdot}142 = 21{\cdot}994 \text{ in. say } 22 \text{ in.}$$

whilst the finished height is 8 in. Draw a rectangle of these dimensions.

(*b*) Add to the rectangle 1½ times the grooving allowance on each end of the long sides, 2½ times the wire diameter at the top and the width of the knocked up joint at the bottom.

(*c*) Notch the corners as shown, the metal on the wired edge being cut away to a depth of 4 times the wire diameter to allow the wire to fit snugly across the top of the grooved joint.

Bottom plate. The outside diameter of the blank must be equal to 7 in. plus 4 times the width of the knocked up joint and spun up so that the bottom dia. is 7 in. plus twice the width of the joint before fitting on to the body.

Note. Suitable allowances for metal thickness should be added to all dimensions if working in medium or heavy gauge material.

Box Making

Fig. 107(a) shows the joint details for a rectangular box 12 in. long × 8 in. wide × 4 in. deep, with $\frac{3}{16}$ in. knocked up ends; ½ in. × 12 S.W.G. wired backlap; hinged lid wired 14 S.W.G., and corner stiffening supports.

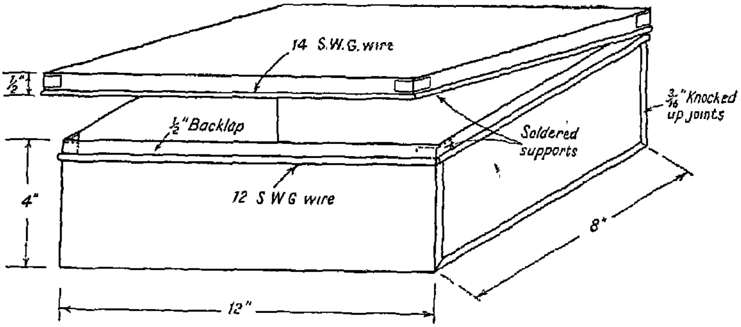


FIG. 107(a)

The overall dimensions of the material may be calculated as follows:

Body

Overall length = Length of box + joint allowance
= $12 + (4 \times \frac{3}{16})$
= $12\frac{3}{4}$ in.

Overall width = Width of box = 8 in.
+ twice depth of box = 8 in.
+ 2, $\frac{1}{2}$ in. backlaps = 1 in.
+ 2 wire allowances = $\frac{1}{2}$ in.
Total Width = $17\frac{1}{2}$ in.

End Pieces (2 off)

Overall length = $8 + (2 \times \frac{3}{16})$ = $8\frac{3}{8}$ in.
Overall depth = Depth of box = 4 in.
+ $\frac{1}{2}$ in. backlap = $\frac{1}{2}$ in.
+ wire allowance = $\frac{1}{4}$ in.
+ joing allowance = $\frac{3}{16}$ in.
Total Depth = $4\frac{15}{16}$ in.

Lid

Overall length = Actual length = 12 in.
+ clearance = $\frac{1}{16}$ in.
+ 2 depths of lid = 1 in.
+ 2 wire allowances = $\frac{3}{8}$ in.
Total Length = $13\frac{7}{16}$ in.

The overall width dimensions may be obtained using similar allowances and substituting 8 in. in the first dimension, when the total width of the top will be found to be $9\frac{7}{16}$ in. The metal should now be cut to these calculated sizes and notched at the corners as shown. Care should also be taken to ensure that notching for the hinges takes place before wiring the metal. Hinge details are illustrated in Fig. 107(b).

By varying the box dimensions all kinds of useful articles may be produced to form sundries such as attaché cases, shaving cabinets, trinket boxes, tool boxes and first aid cabinets. As well as forming a useful part of the training of the student these will be found to be most stimulating exercises.

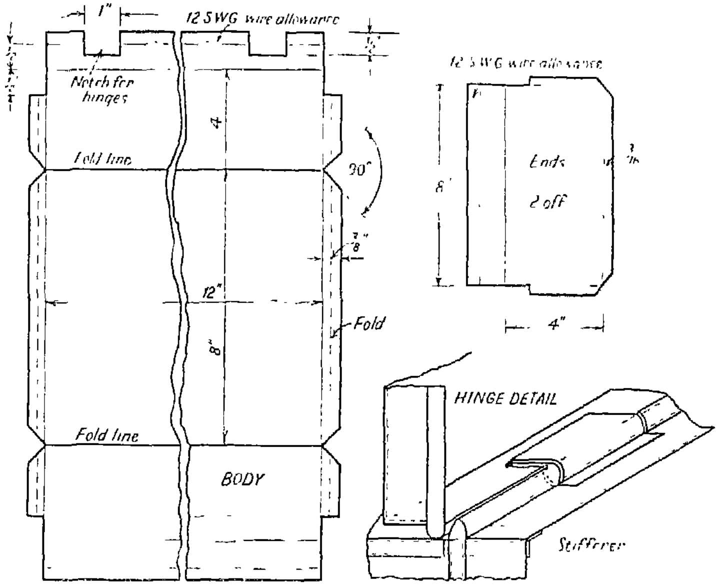


FIG. 107(b)

Allowance for Peened down Edges

Where a double edge for a peened down joint is required in connection with cylindrical articles, such as the joining of lobster back segments, it will be found that in spinning off or flanging the allowance the joint tends to open and form a Vee on the flanged edge (Fig. 108(a)). The reason for this is that there is little or no resistance to the metal opening in the vicinity of the joint. To prevent its occurrence, stretching of the metal should always be begun in the area near the joint line and thus the metal becomes stiffened.

When the article is made of thin metal the excessive hammering required to close the Vee may, in turn, so thin the metal that cracks are formed round the edges. It is however possible to cut the metal in such a way that no excessive hammering is required.

Consider the arrangement where a sheet is marked out to give the allowances for a vertical grooved joint and a peened joint along the top edge (Fig. 108(b)).

The dotted portion is not cut away altogether, but only along the inner line to the depth of the grooving allowance. This will allow the groove to be formed in such a way that the peening allowances will overlap when the metal is rolled into its cylindrical form (Fig. 108(c)). When the flanging of the edges is done there will be sufficient metal available to prevent the Vee forming. Any surplus metal can be removed by cutting from the outside to the inside of the flange, when the excess will fall away (Fig. 108(d)).

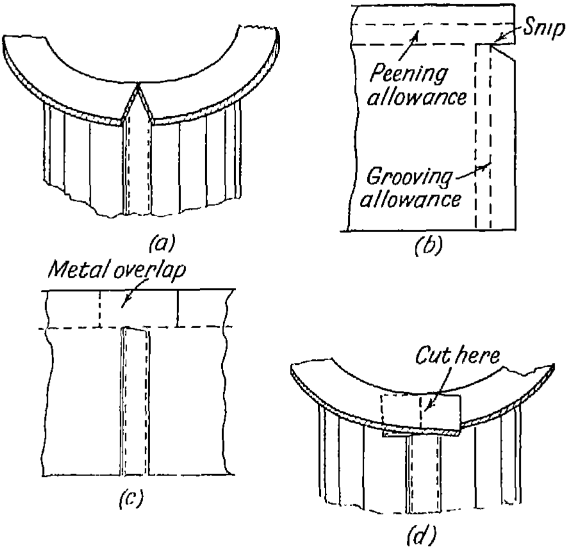


Fig. 108

Bending Allowances

When a piece of metal is bent it is subjected to two distinct stresses. The metal on the outside curve is stretched while that on the inside curve is compressed. At the centre of the thickness the metal is neither in tension nor compression since on either side there is an opposite stress. This central plane of unstressed metal is called the *neutral axis* or *neutral line* (Fig. 109).

The length of the metal piece along this line is the same as before bending, the outside surface becomes longer and the inside shorter. If thick metal is bent the part in tension may split and the inside curved part may buckle and wrinkle. For this reason thick sections are bent hot because the hot metal is more plastic. Bending thin metal will create less stress but it is important to realise that these stresses are still present. The smaller the radius of bend the greater the stress.

Where the radius of the bend is large when compared to the metal thickness, it may be assumed for all practical purposes that the neutral line will lie at the mid-point between the surfaces of the metal. If a piece of flat sheet, say $\frac{1}{8}$ in. thick and 31·42 in. long, is rolled to form a circle, it will be found that the finished diameter will be 10 in. plus $\frac{1}{8}$ in. on the outside and 10 in. minus $\frac{1}{8}$ in. on the inside. Therefore, the following measurements can be taken from the formed circle:

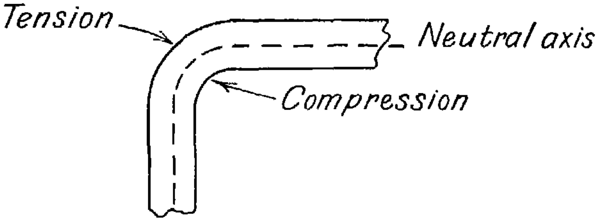


FIG. 109

Inside diameter $10 - \frac{1}{8} = 9\frac{7}{8}$ in.

Outside diameter $10 + \frac{1}{8} = 10\frac{1}{8}$ in.

Neutral diameter $= 10$ in.

Hence the allowance for the metal thickness must be taken into consideration when one article must fit either inside or outside another. In general this allowance is π times the metal thickness to be added to or subtracted from the mean circumference.

EXAMPLE 1. *Determine the length of material required to form a stove pipe* 4 *in. inside dia. if the thickness of metal is* 20 *S.W.G.*

Circumference = 4 × 3·14 in.
= 12·56 in.
Metal allowance = 3·14 × 0·036
= 0·113
Total length of material = 12·56 + 0·113
= 12·673 in.
= 12·67 in. approx.

EXAMPLE 2. *Calculate the length of material* (14 *S.W.G.*) *required to make a tube of* 9 *in. outside diameter and also a ring support* $\frac{1}{8}$ *in. thick to fit over the outside of the end.*

Circumference of tube = 9 × 3·14 in.
= 28·26 in.
Metal allowance = 3·14 × 0·08
= 0·251
Total length for tube = 28·26 − 0·251
= 28·009 in.
= 28 in. approx.
Inside circumference of ring = 3·14 × 9
= 28·26 in.
Metal allowance = 3·14 × 0·125
= 0·3925 in.
Total length for ring = 28·26 + 0·3925
= 28·6525 in.
= 28·65 in. approx.

Note that the constant π has been taken as 3·14 and the answers are to the nearest 1/100th of an inch. In general the following formulae will be found suitable for most practical purposes.

When D is the outside diameter required and t is the metal thickness,

$$\text{length of material to be cut} = \pi(D - t).$$

When D_1 is inside diameter required,

$$\text{length to be cut} = \pi(D_1 + t).$$

When a series of metal tubes are to be made so that they interlock with each other and so form a long tubular article, then the difference in the length of material required between the inside circumference and the outside circumference of the mating surfaces must be at least 6·284 times the metal thickness. This would give a perfect fit if the metal could be rolled into an exact circle, but this is not possible in practice and it is therefore usual to make an allowance of 7 times the metal thickness to ensure some clearance.

Blank Diameters for Cold Working

The method suggested assumes that the surface areas of the shell and the blank must be equal.

Hemispherical Bowls

Surface area of bowl = surface area of blank

$$\therefore \qquad 2\pi r^2 = \pi R^2$$

where r is radius of bowl and R is radius of blank.

Dividing both sides of the equation by π

$$\begin{aligned} 2r^2 &= R^2 \\ \text{From which} \qquad R &= \sqrt{2r^2} \\ R &= \sqrt{(2)} \times r \\ R &= 1{\cdot}414r \end{aligned}$$

EXAMPLE 3. *Determine the blank diameter required to raise a hemispherical bowl of 6 in. diameter.*

$$\begin{aligned} &= \sqrt{(2)} \times r \\ \text{Radius of blank} &= \sqrt{(2)} \times 3 \text{ in.} \\ R &= 1{\cdot}414 \times 3 \text{ in.} \\ R &= 4{\cdot}242 \text{ in.} \\ \text{Diameter of blank} &= 8{\cdot}482 \text{ in.} \\ &\underline{\text{say } 8{\cdot}48 \text{ in.}} \end{aligned}$$

EXAMPLE 4. *A metal disc 10 in. diameter is to be formed into a hemisphere. Calculate the diameter of the hemisphere.*

$$\begin{aligned} &= \sqrt{(2)} \times \\ \text{Now, } R &= 1{\cdot}414r \\ 5 &= 1{\cdot}414r \\ \text{From which } r = \frac{5}{1.414} &= 3{\cdot}537 \text{ in.} \\ \therefore \quad \text{Diameter of hemisphere} &= 7{\cdot}074 \text{ in.} \\ &\underline{\text{say } 7{\cdot}07 \text{ in.}} \end{aligned}$$

These calculations may be entirely eliminated in practice by simply drawing a semicircle to represent the cross-section of the bowl (Fig. 110) and measuring the diagonal distance between a point A on the circumference which is vertically above the centre point and point B which is shown on the base. The length AB will be found to be $\sqrt{(2)} \times r$ where r is the radius of the semicircle.

Cylinders

Similar calculations may be used to derive a formula for the blank size required to form a one-piece cylindrical shell.

Surface area of cylindrical shell = surface area of blank

$$\therefore \qquad \pi r^2 + 2\pi rh = \pi R^2$$

where r is the radius of the cylinder,

h is the height of the cylinder,

R is the radius of the blank.

Dividing through by π

$$r^2 + 2rh = R^2$$

from which

$$R = \sqrt{(r^2 + 2rh)}$$

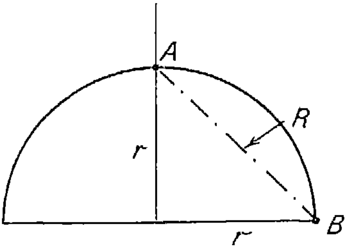


FIG. 110

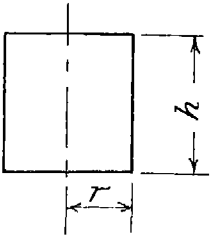


FIG. 111

EXAMPLE 5. *Determine the diameter of blank to produce a one-piece cylindrical vessel 4 in. in diameter, 2 in. deep.*

$$R = \sqrt{(r^2 + 2rh)} = \sqrt{(4 + 8)} = \sqrt{12}$$

$$\therefore \quad R = 3{\cdot}464$$

Diameter of blank = 6·928 in.

say 6·93 in.

EXAMPLE 6. *Calculate the height of a cylindrical vessel made in one piece from a 10-in. diameter blank if the radius of the cylinder must not exceed* $1\frac{1}{2}$ *in.*

$$\begin{aligned}
\text{Now } R^2 &= r^2 + 2rh \\
\therefore \quad 5^2 &= 1\tfrac{1}{2}^2 + 2 \times 1\tfrac{1}{2}h \\
25 &= 2{\cdot}25 + 3h \\
25 - 2{\cdot}25 &= 3h \\
22{\cdot}75 &= 3h \\
\frac{22{\cdot}75}{3} &= h \\
7{\cdot}583 \text{ in.} &= h
\end{aligned}$$

say 7·58 in.

Unlike the hemisphere, the calculations required for determining the blank size of a cylindrical shell cannot be replaced by diagrammatic illustrations and therefore this is the only method by which an accurate blank diameter can be obtained.

Annealing

When metals are worked into shapes by hammering, rolling or wheeling, they become work hardened and a state of stress is set up within the metal. These internal stresses are removed by a process known as annealing, which consists of heating the metal to a predetermined temperature and then allowing it to cool. The method of cooling is, in many cases, the critical factor that governs the softness of the annealed metal. The period of heating must not be longer than necessary, for overheating will cause the grains of the metal to become fewer in number and larger in size. This phenomenon is known as *grain growth* and reduces the tensile strength and resistance to impact.

Methods of Annealing

Steels. The temperature at which the crystals of steel rearrange themselves varies with the carbon content of the metal. In the case of low carbon steels, which are used mainly in sheet metal work, the heating temperature is approximately 900°C, but in the annealing of high carbon steels a temperature of 750°C only is required. A suitable temperature for other steels may be found by drawing the simple graph given in Fig. 112, where the carbon percentages are plotted on the horizontal base line, and the temperatures on the vertical scale line. For pure iron the critical temperature is 900°C, while for 0·9% carbon steel the temperature required is only 700°C. These points are joined together by the straight line AB, which gives the annealing temperature of any steel within that range.

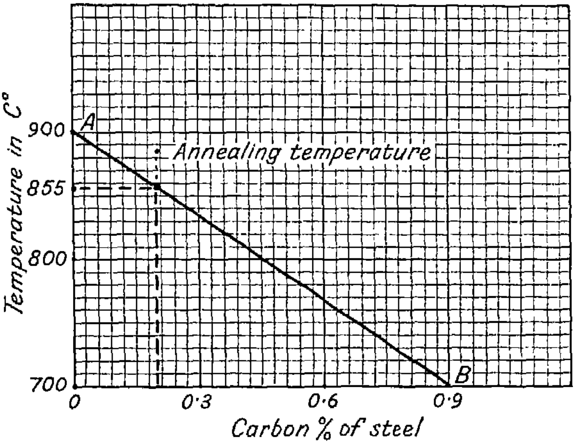


FIG. 112

It is usual to add 30°C to the temperature found from the graph to allow for any impurities in the metals. For instance, if the annealing temperature of a 0·2 carbon steel is required, a vertical line is drawn from 0·2 on the horizontal scale to cut through the line AB. At the point of intersection a horizontal line is drawn across to the temperature scale and the correct annealing temperature will be obtained. 30°C is added and the annealing temperature for this steel is approximately

$$855°\text{C} + 30°\text{C} = 885°\text{C}.$$

Rapid cooling may cause small cracks to occur in the metal. It is, therefore, advisable to cool steel slowly after heating, if maximum softness is to be obtained.

Copper. The metal should be heated slowly to 500°C, a dull red heat, and maintained at this temperature for a short time to allow the crystal arrangement to take place within the metal. At temperatures higher than 500°C grain growth becomes rapid and therefore the metal should not be overheated. Copper is cooled by quenching in cold water; this also has the effect of cleaning the metal, giving it a bright red appearance. The rate of cooling is not, however, critical as far as the annealing process is concerned, and the metal may be allowed to cool slowly if desired. If this is done, a film of black oxide will form on the surface of the metal. The oxide is brittle and can easily be removed by brushing when cold. Copper which has been tarnished or is otherwise dirty may have its surface brightened by dipping in a bath of dilute sulphuric acid containing 1 part acid and 70 parts of water.

Brass (see p. 131)

Aluminium. The best results are obtained by heating aluminium sheets to 350°C and then cooling slowly, or where time is important by quenching in water. For normal sheet metal parts a large gas or electric muffle furnace is employed to supply the heat, and this must be thermostatically controlled. When this is not possible, because of the size of the part for example, the heat is supplied from a gas and air blowpipe.

Aluminium does not show any visible change of colour when heated and therefore it is difficult to estimate the correct annealing temperature. In practice, the following procedures are adopted:

(*a*) Smear the metal with machine oil and heat until the oil is burnt away.

(*b*) Heat the metal slowly until if scratched with a wooden splinter a dark brown stain appears on the surface. A match stick is ideal for this.

(*c*) Heat until the metal is stained black when smeared with a piece of soap.

Zinc. No method has yet been discovered whereby zinc can be annealed and maintained in this condition at low temperatures. The metal can, however, be made easier to work by heating to 100°C and using it at that temperature. On cooling the metal will become brittle again. This explains why zinc sheets should never be worked when completely cold.

Rivets and Riveting

Rivets are used for connecting members of structural frames, and in the fabrication of tanks, boilers, containers, etc. Manufactured rivets consist of a cylindrical portion known as the *shank* or *body*, and a *head* which is integral with the body. This is technically known as the manufactured head; the part formed by the workman to fasten the joint is known as the driven head.

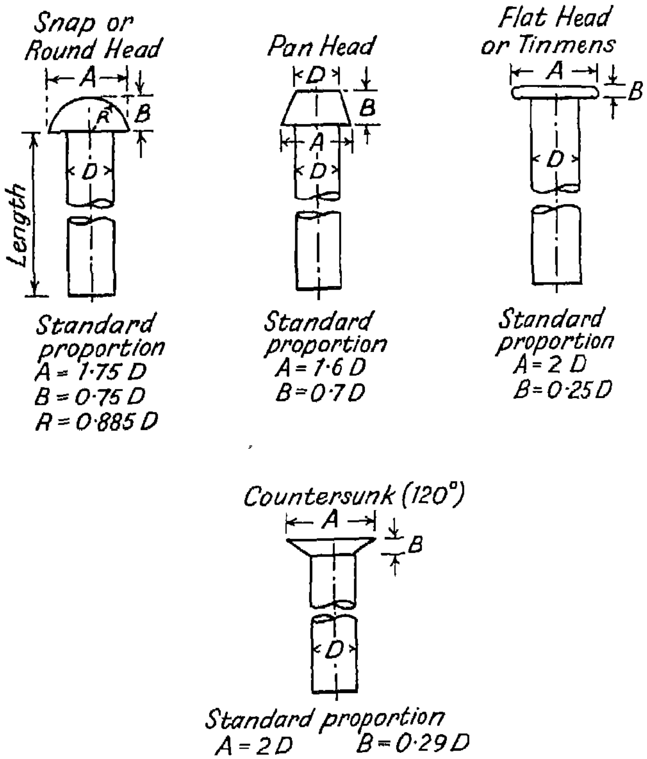


FIG. 113

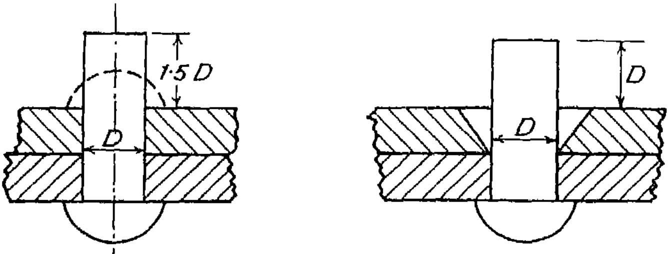
Rivets are designated by the diameter, length and type of head, e.g., $\frac{5}{8} \times \frac{1}{4}$ in. tinners flat rivet. The heads are of various forms, several of which are shown in Fig. 113.

The total length of any rivet will depend on two main factors:

(*a*) Its grip, which is the total width of the sheets or plates held together.

(*b*) The shape of the driven head. The total length of projection required for riveting is proportional to the rivet diameter. The allowance for countersunk rivets is approximately equal to the rivet diameter, and for raised heads a projection equal to $1\frac{1}{2}$ times the rivet diameter is required. To determine the correct diameter of any rivet the following formula may be used:

$$\text{Diameter of rivet} = 1{\cdot}2\sqrt{\text{thickness of plate}}$$



Allowance for Raised Heads

Allowance for Countersunk Heads

FIG. 114

Overlap of plates. The distance between the centre of one rivet hole and the edge of the plate must never be less than $1\frac{1}{2}$ times the diameter of the rivet. Hence, for a single lap joint the overlap will be 3 times the diameter of the rivet.

Rivet spacing. The spacing of rivet holes in light sheet metal work varies with the type of component, but in general the space should not be less than 3 diameters of rivet, although in special circumstances this may be extended to 8 diameters.

Methods of preparing joints. Holes should preferably be drilled and all sharp edges removed. Where several plates are required they should be clamped together and drilled as one. In sheet metal work it is, however, more usual to obtain the holes by punching with either a mechanical or hand-operated punch. If the latter is used a small jig can be made by folding a piece of strip at right angles and fastening on the surface a rivet of the desired size; in this way a constant pitch between the holes will be assured (Fig. 115).

Where extremely thin material is used the drilling or punching of holes is omitted entirely, and with the aid of a rivet set the rivets are drawn through the solid plate in any desired position. A rivet set is a special tool with a thin shank and a rectangular face. The face is drilled with a clearance drill to enable the rivet to pass into it when the top of the shank receives a sharp hammer blow. Also on the face is an indentation of the shape of the rivet head. This is then placed on the head of the rivet whilst more hammer blows descend on the shank, thus sealing the joint and forming the driven head.

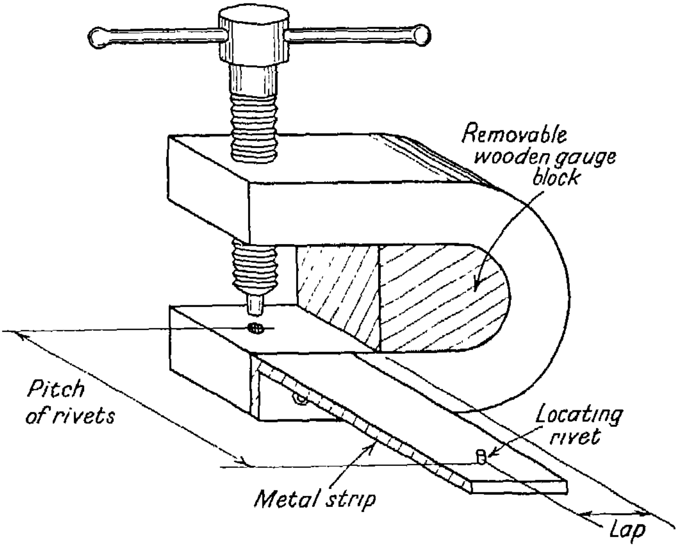
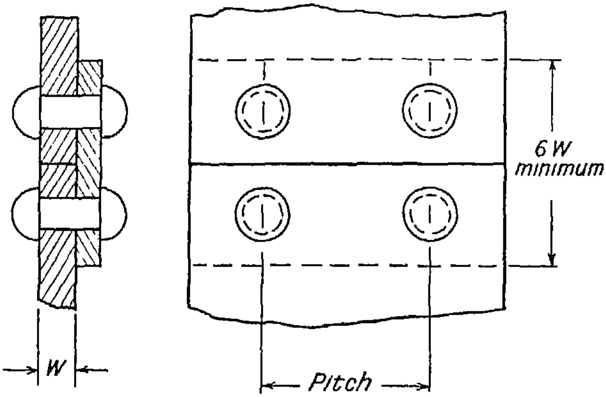


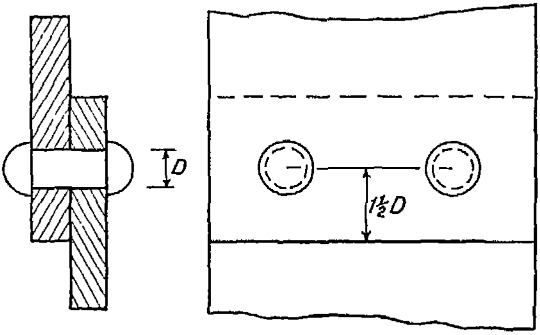
FIG. 115

Types of joints. Fig. 116 shows a section of a **butt joint** between two flat plates. In all joints of this type a supporting plate known as a *strap* must be placed on one or both sides of the metal, and the width of the strap should never be less than 6 times the rivet diameter. Where overlapping of the plates is permissible, a single riveted **lap joint** (Fig. 117) is used. This is the type of joint most frequently employed by sheet metal workers and is used for circular work and air duct pipe connections, as well as for flat sheets. If desired, double riveted lap joints (Fig. 118), with rivets placed in sequence or in a zig-zag manner, may be used as alternatives, but generally these are reserved for heavier work, such as boiler making.



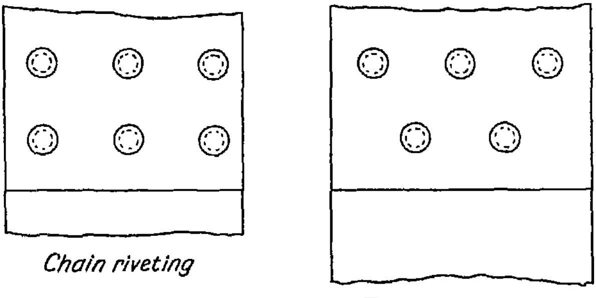
Single lap butt joint

Fig. 116



Single lap joint

Fig. 117



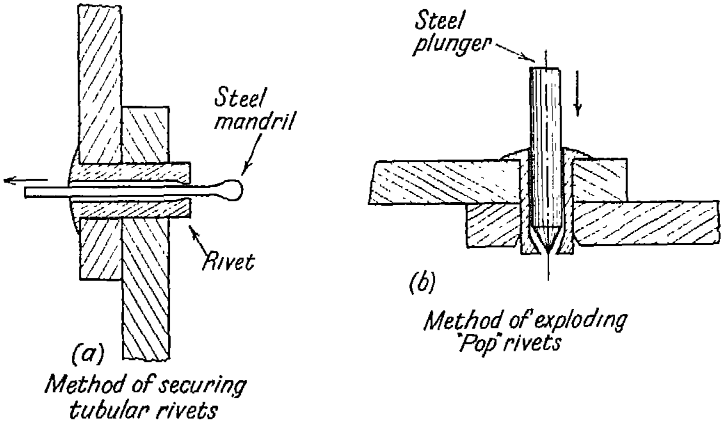
Chain riveting

Zig-zag riveting

Fig. 118

Modes of failure. The main forces acting on a riveted joint are shearing forces, which tend to cut the rivet across its diameter or force apart the metal between the rivet and the edge of the plate. To minimise this possibility, the overlap of the plates and also the rivet diameter must not be less than those already suggested.

Failure through the rivet shearing may be accelerated by drilled or punched holes not being aligned truly, and by forcing the rivet into the holes. The rivet body becomes distorted and in many cases reduced in sectional area, hence the shearing force required to shear the rivet will be considerably lower than normal. This mode of failure cannot be overcome by re-drilling the holes and using a larger rivet, for this will lead to a condition where the strength of the rivet is considerably more than that of the plate, and the latter will become crushed when exposed to heavy loads. Should this occur the rivet will work loose and ultimately fall out of the hole.



(a) Method of securing tubular rivets

(b) Method of exploding 'Pop' rivets

FIG. 119

A common failure of riveted work is the bursting of the plates between the rivet holes or the edge of the material. The former is caused by placing rivets too close together and the latter by incorrect overlaps on the plate. If a riveted joint is to have a maximum efficiency it should be designed so that, when tested until breaking load is reached, the modes of failure of such a joint occur simultaneously.

Special rivets. In the motor car and aircraft industries riveted joints are placed in such positions that the possibility of failure through excessive loading is remote, and therefore some latitude is allowed in both the design of the joints and also on the type of rivets used. Tubular rivets (Fig. 119(a)) are often employed in aircraft construction and possess the advantage of extreme lightness combined with a larger bearing area for a given shear strength. Special riveting machines are used to form the head, and one of the chief disadvantages of this type of rivet is that the formed head does not hold the sheets firmly when exposed to shock loads. In many cases the rivets are manufactured with a tapered bore and are expanded into position by drawing a hardened steel mandril through the hole. A specially designed rivet gun has been developed to enable one side riveting of sheet metal parts to be carried out. The process is a continuous one. The machine places the rivet in the hole and withdraws the mandril. On its return the mandril enlarges the hole in the rivet and forms the head on the ' blind side ', thus enabling the operation to be carried out speedily and rapidly. There are two main types of tapered tubular rivets.

(*a*) One in which the mandril is used continuously and therefore does not require replacement, or another type where the mandril is fixed in the rivet and when a predetermined pressure is applied by the machine the mandril breaks and leaves a solid rivet. If required, the broken mandril may be removed by a sharp pointed extractor.

(*b*) Pop rivets are another type of hollow rivet which may be expanded from the inside and are used extensively for joining aluminium alloy sheets together. It will be seen from Fig. 119(b) that a hardened conical mandril is forced through the rivet, thus causing it to expand and also form the driven head.

Hollowing

Metals which possess good malleability and ductility can be formed into complicated shapes of double curvature by the process of hollowing. The work is held in position on a wooden block or sand bag, and beaten into shape using a round headed hammer or mallet. For the process to be carried out successfully, the hammer blows should fall on the same spot on the block, and the work be rotated or moved so that each part receives an even blow. Work should always commence on the outside edge and be continued spirally until the centre is reached.

In all hollowed work it is essential for annealing to be carried out at frequent intervals, to prevent cracking, due to the metal becoming work hardened. Copper, brass and aluminium are the metals most suitable for hollowing, but mild steel, zinc and tinplate can be successfully manipulated. Coated metals can only withstand a small amount of hollowing, as it is not possible to anneal them without destroying the protective surface coating.

Hollowing tends to thin the metal and also stretch it; care must be taken on work of small radii to ensure that the metal does not burst under the force of the hammer blows. The fracture will usually occur in the centre of the disc where the stress is at a maximum, and only practical experience will enable the craftsman to recognise the approaching danger, for conditions differ with each material.

It can be readily seen that hollowing of deep articles tends to weaken the metal, and the process is therefore limited to shallow vessels such as boiler ends, dome tops and curved lids. Where large quantities of a product are required to uniform dimensions, hollowing has been superseded by pressing. But this is only done when the quantity required warrants the cost of the press tools, which are expensive to produce.

Templates. Before work is begun on any article to be hollowed, a template of the same shape as the cross-section of the finished job is made. Where the work forms part of a hemisphere a flat plate cut to the required diameter will suffice, but when the work involves more complex curves it may be necessary to produce more than one template. Beginners are advised to check the work against the template after each course of hollowing and to rectify any mistakes, for if these are left to accumulate the ultimate results may prove to be disastrous. (Many a piece of metal has been cut out to form a hemisphere, and finished in the form of a piece of scrap.)

Blocking out. This is another form of hollowing and is carried out by beating the metal on the open end of a thick hollow steel cylinder. The end is usually reinforced by an angle iron frame bent to form a circle of inside diameter equal to that required for the finished article. The process may be performed whilst the metal is hot, but this is usual only in the case of mild steel and, to a lesser extent, thick copper sheets.

A coke fire, driven by compressed air, is used to heat a small area of the metal to bright red. The metal is held over the ring and beaten to the shape required. The metal is then returned to the fire and the process is repeated over another small area adjacent to the first one. This procedure is continued until the whole of the disc has been formed to shape. A large wooden mallet is used for forming the metal, which is not supported underneath. Two operators are required, one to hold the work with two pairs of tongs, and one to carry out the beating. Care must again be taken in the malleting to ensure that the metal does not burst, for at high temperatures the tensile strength of the metal is considerably reduced.

When a metal is hammered to alter its shape without any support underneath, the workshop name to describe this process is ' hammering on the wind '.

Raising

The results produced by hollowing can also be obtained by a process known as ' raising '. The metal disc is held over the end of a parallel bar or forming head and beaten into shape, using a mallet which may be round headed or cut to form a chisel point. As opposed to hollowing, the beating is started in the centre of the work and continued by revolving the workpiece spirally until the outside edge has been reached. The metal must not lie flat on the bar as it is placed below the falling mallet, but should be inclined at a slight angle to it. In this way the impact force will drive the metal inwards and hence raise the central portion upwards and outwards. No thinning of metal can take place during the process and the metal tends to thicken on the outside edges; thus the possibility of the metal bursting is entirely eliminated.

The chief difficulty encountered in raising is the tendency of the edges of the material to overlap each other as they are forced inwards, and hence the disc is reduced in circumference. These difficulties only arise if the process is carried out too quickly, and they can be overcome by reducing the angle between the bar and the metal. As in the process of hollowing, the metal must be annealed after each course if satisfactory results are to be obtained.

A combination of hollowing and raising is often employed to produce articles at a quicker rate, and if care is taken a course by each method can be carried out between annealing. When, say, a complete sphere is to be made with thin material, two discs are often fastened together and worked up as one and then parted on completion of the shape of a hemisphere. The advantages of this are:

(*a*) The time of the operation is reduced by half.

(*b*) The two halves of the sphere will be identical and easily jointed in the middle.

Like hollowing, the process of raising simple shapes is being superseded by presses. Also, many articles once formed by hand are now produced by a process known as ' spinning '. This, briefly, consists of holding the workpiece in a former attached to a high speed lathe headstock and forcing the metal into shape with the aid of form tools. Spinning is now used for batch production of many hollow articles.

Methods of Restoring an Even Surface

When articles have been produced by working processes, such as hollowing or raising, their surfaces tend to be uneven. To remove these blemishes a further operation has to be carried out, and in all cases before commencing the treatment a final annealing is required to restore the metal to its normal state.

Planishing is essentially a process of hammering a part when supported underneath by a solid hard surface of the same curvature. The hammering may be done either by hand or by a machine driven unit, but the aims are the same in both cases:

(*a*) To close the grain of the annealed metal.
(*b*) To harden the metal and enable it to keep its shape when exposed to shock loads.
(*c*) To give a smooth finish.

A planishing hammer usually has a round head with a slightly convex face. The curvature of the face is so slight that it can hardly be seen, but nevertheless it is essential; it must be clean and well polished. Planishing heads are made of hardened steel or cast iron; these also should be highly polished (see Fig. 91). The heads may be held firmly in position beneath the work by hand or, if circumstances permit, by a fixture.

Each hammer blow should be directed in such a way that it slightly overlaps the adjacent ones, giving the workpiece an even degree of hardness throughout its mass. The surfaces of copper and brass articles quickly show the marks left by the hammer, which are somewhat darker than the soft unhammered metal. These marks enable one to see with comparative ease the area that has been completed; hence no difficulty is encountered in defining the softer areas from those areas which have been planished, if the work is not carried out at one time.

Mild steel does not show the hammer marks quite so clearly as the non-ferrous metals and for this reason the workpiece is often coated with a solution of black lead and water and then allowed to dry. When the subsequent planishing is carried out the black lead on the surface of the steel becomes highly polished, thus enabling the worker to see at a glance how the operation is progressing.

Planishing by hand is a slow arduous task; it is only carried out on small articles, except in exceptional circumstances, and modern methods of production are being introduced at every opportunity to eliminate still further the process.

Wheeling is another method used to restore an even surface to articles that have been distorted by hollowing or raising, although this is not the only use of the process. In many instances it is possible to produce components of double curvature from flat sheets by ingenious use of the wheeling machine.

The machine consists of a cast iron frame bolted on to a base, the overall height being about 6 ft and the weight 11½ cwt. The top wheel is fixed and rotates on ball bearing runners fastened into the cast iron frame; a spindle which is attached to the wheel and carries the operating handle is supported by the bearings. A lower wheel of smaller dimensions is carried in a steel bracket known as the head, this being operated to give vertical movement to the wheel by means of a small foot-operated turntable. Easy removal of the lower wheels is important, for these are changeable; the rims may be flat or curved depending upon the shape of the work and also whether the operation is ‘ smoothing ’ or shaping.

To carry out the wheeling process the head is lowered and a suitably shaped wheel fitted into position. The workpiece is then placed into position between the fixed top wheel and the movable lower one, slight pressure being applied to the surface of the sheet by means of the foot-operated turntable. The work is guided backwards and forwards through the wheels as the operating handle is traversed to and fro. The operation is purely one of cold rolling and subsequent work hardening. With practice the smoothing operation can be successfully mastered and applied to most materials. The only real difficulty is that of estimating the correct pressure between the wheels, which varies for each metal, and also ensuring that all the areas of metal pass under the wheels the same number of times, thus obtaining an evenly finished surface. When wheeling is carried out on hemispherical articles it is important that the runs do not terminate at the centre, otherwise the shape will become distorted. The work should be divided into small areas and each treated in turn, and passed through the rolls approximately the same number of times.

Wheeling machines have also been adapted for the production of motor car body components, where steel sheets are formed into sharp curves of double radii. The base wheel is shaped to the curve required and the sheets are welded together at right angles along an edge cut to the shape of the secondary curve.

The welded joint is then passed through the machine, whilst high pressure is applied to the rolls, these being mechanically rotated. The action is twofold: the desired radius is formed on the component, and the weld is crushed until it is the same thickness as the plate and therefore does not require grinding. Many components which used to be pressed and required expensive press tools are now produced in this way.

Stretching and Flanging

In stretching a metal, such as throwing off a flange on the end of a cylindrical article, the metal should not be flat on the surface of the stake but be inclined at a slight angle to it, thus enabling the metal to be drawn over as well as stretched. The hammer used for this operation is the stretching hammer (Fig. 90). It has a narrow rectangular body with rounded faces to prevent cutting the surface of the material during the process. Each blow should force the metal downwards and outwards in such a way that the outside is stretched, hence ensuring that there is sufficient material to give the required increase of circumference at the outer edges of the flange.

Unfortunately, stretching tends to thin the metal and also harden it. Care must therefore be taken that cracking does not take place at the outer edges. To reduce this possibility, slow working with an even, steady blow and repeated annealing is required. A good practical tip is to planish the metal occasionally between courses of stretching. The action of this is twofold; it removes the indentations made by the stretching hammer and also ensures an even degree of hardness of the metal being worked, thus reducing the tendency to crack.

Stretching may be carried out while the metal is hot on both mild steel and copper if desired. But aluminium, brass, zinc, and all coated metals should only be worked when cold. It is also advisable to draw file the edges and make them smooth before starting the process, otherwise the metal will certainly crack as soon as any working is attempted.

Flattening

The object of this process is to relieve locked up stresses in a metal and prevent later distortion. In the manufacture of metal sheets all buckles are removed by machine, but cutting, bending and working of any kind tends to restore them, so a certain amount of flattening by hammer is done in the workshop. The operation is by no means easy and calls for a high degree of skill and experience, which can only be obtained by practice, for the effect of every hammer blow must be carefully watched and its effect mentally assessed. For instance, if a sheet is straight along the edges and has a centre buckle, it is obvious that there is more metal in the centre and the edges are in tension. To relieve this condition, and hence flatten the plate, the outer edges require hammering heavily and the intermediate areas, between the centre buckle and the edges, comparatively lightly. If this is carried out correctly a number of small buckles will be seen to form over various parts of the plate. These are removed by a series of light blows directed on the plate, care being taken not to hit the buckles but the portions surrounding these areas. Finally, when the sheet is comparatively flat the whole plate is hammered lightly with an even, steady series of blows. A good method is to start at one corner and work diagonally across the sheet. The metal must be turned over at frequent intervals and the operation performed on both sides.

Should a plate be flat in the centre and the edges buckled, the reverse of the above procedure is carried out, i.e., stretch the inside and work slowly to the edges. Flattening can only be successfully accomplished if the hammer has a level face, slightly bevelled at the edges, and the workbench is perfectly flat and solid. A thick cast iron marking out table is ideal for this purpose.

Soft Soldering

Soldering is the joining of two metal surfaces by means of a solder which has a lower melting point than the metals being joined. It is more than simply cementing or sticking the metals together. In a correctly soldered joint a definite chemical reaction takes place, the solder forms an intermetallic compound with the parent metal and therefore becomes, to some extent, a part of it. The layer of compound will grow deeper if the joint is kept at the soldering temperature.

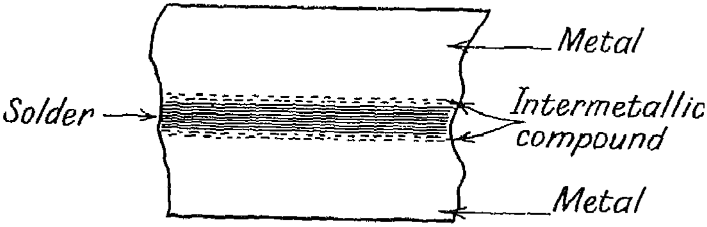


FIG. 120

This action can only take place however if the two surfaces to be joined are *chemically clean.* That is, they must not only be free of dirt and grease but also of any oxide film due to the surfaces being in contact with the air. If solder is applied to a metal surface which is not chemically clean it will run into beads or globules and will not ‘ wet ’ the surface. There is then no true contact between the solder and metal due to the skin of oxide on the metal surface and on the solder. If this skin is removed the solder will flow freely, spread thinly over the surface and wet it. Fortunately this apparently difficult operation is done quite simply by using a suitable flux.

The joint is further strengthened by the very intimate contact of the solder and parent metal which causes what may be called ' inter-atomic adhesion '. The molecules of the metals attract each other and resist any tendency to pull them apart.

There are then four essentials:

(1) The parent metal and solder must be capable of forming an intermetallic compound.
(2) The correct flux must be used so that the surfaces will be chemically clean and the solder will wet the surfaces.
(3) The joint must be such that the molten solder can flow between the joint faces and completely fill the space.
(4) Sufficient heat must be applied to make the solder flow freely.

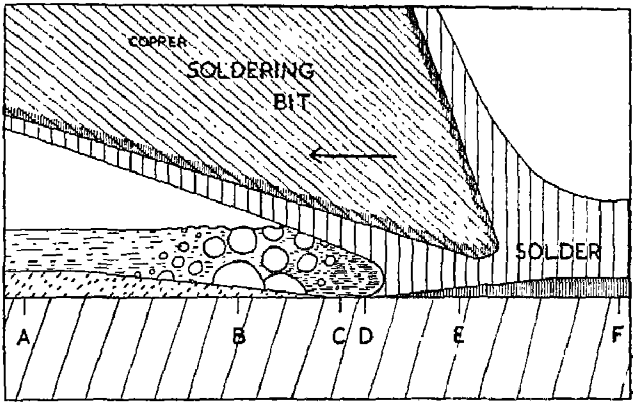


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FIG. 121. Diagrammatic representation of the displacement of flux by molten solder.

A. Flux solution lying above oxidised metal surface.
B. Boiling flux solution removing the film of oxide.
C. Bare metal in contact with fused flux.
D. Liquid solder displacing fused flux.
E. Tin reacting with the basis metal to form compound.
F. Solder solidifying.

Cleaning and the Flux

New sheets of the coated metals, such as tinplate, terneplate or galvanised iron do not require cleaning, except perhaps degreasing. This is most effectively done by immersion in trichlorethylene vapour or other spirit solvents. Other metals are cleaned by scraping, filing, shot blasting or scratch brushing. Rubbing with steel wool is also very effective.

Rust can quickly be removed from mild steel with hydrochloric acid. This may be used either concentrated or as a 50 per cent.

solution in water. The concentrated acid is normally used for small parts and should be used with great care as its action is very rapid and it will quickly pit the surface. The diluted solution allows greater latitude and even with this the parts should be immersed only for a short time, possibly up to 15 min. The parts are then washed in warm water to remove the acid and dried. It is obviously very important to ensure that all the acid is removed. This pickling process can also be done with dilute sulphuric acid.

Fluxes may be classified as active or inactive. The active fluxes attack and dissolve the oxide film on the metal and also act as a barrier to prevent further oxidisation. The inactive ones protect a previously cleaned surface and have little effect on oxide. Fig. 121 illustrates how the flux removes the oxide, protects the surface and then gives way to the molten solder to form the joint.

The Active Fluxes

These are the more efficient and are normally used in sheet metal work.

‘ Killed spirit ’ (Zinc chloride) is made by dropping zinc clippings into spirits of salt (hydrochloric acid) or by dissolving 6 ounces of fused zinc chloride in one pint of water.

In use the killed spirit boils, the water evaporates leaving a more concentrated solution which now attacks and removes the oxide. The flux now forms a protective bubble over the surface until the molten solder displaces it to make the joint.

When the joint has cooled the flux leaves a crust close to the joint and this if left would cause rapid corrosion. It can be removed by washing in hot water containing a few drops of hydrochloric acid per gallon and rinsing in hot water having some washing soda dissolved in it to neutralise the acid.

If only tap water is available the part should be left for a few hours; the chloride absorbs moisture from the air and can now be washed clean in hot tap water.

The high melting temperature of zinc chloride (262°C) can be reduced by adding 10 per cent. of ammonium chloride. This is desirable whenever the soldering temperature must be kept low and there is the danger of unmelted crystals preventing a completely soldered joint.

Ammonium chloride. This solution is much less effective and its corrosive after-effects are as serious as those of zinc chloride. It is sometimes used for the tinning of soldering bits. The vapour is poisonous and should not be inhaled.

Hydrochloric acid is used at half strength for soldering zinc since it is more effective in cleaning than is zinc chloride. A mixture of this acid and zinc chloride in equal proportions is a useful flux for stainless steel. The vapour from this flux is dangerous and should not be inhaled. The condensed vapour and spray are extremely corrosive and the whole article must be thoroughly washed after it is soldered.

Phosphoric acid. A solution of orthophosphoric acid, H_3PO_4, 40 per cent. by volume, is an effective flux for steel, copper and brass. It leaves a glassy residue which on steel seems to act as a protective coating. In some conditions it is corrosive and should not therefore be considered as a ' safe ' flux.

The Inactive Fluxes

These are made from resin, a gum which is exuded (or oozes out) from wounds in the bark of pine and fir trees. It is also called rosin or colophony and chemically is abietic acid and related substances. At ordinary temperatures it is solid and melts at about 125°C; at soldering temperature it reacts mildly. It may be brushed on as a solution, sprinkled on the joint or applied as a prepared paste.

The solution is made by dissolving rosin in methylated spirit or industrial spirit. One part of rosin to two to four parts of spirit by weight are put into a long necked flask without a stopper. The process can be accelerated by standing the flask in hot water. (It must never be heated over a flame as this may cause a fire or explosion.)

Rosin flux may be made more active, and of course more corrosive, by adding oleic or lactic acid. Suitable composition are:

Rosin	20 per cent
Oleic acid or Lactic acid	5 per cent
Methylated spirt	remainder

Oleic acid is only moderately effective on steel and its slow action and offensive odour make it unpopular. It is sometimes used for containers which must be made with a flux free from chlorides.

The Soldering Bit

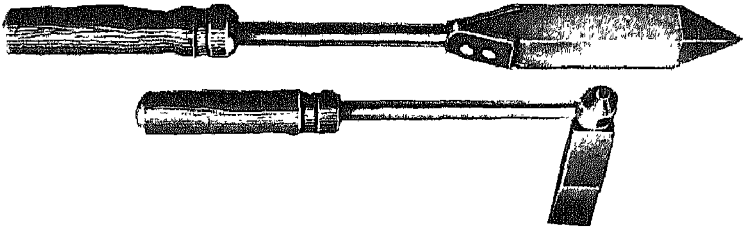
A hand soldering bit consists of a copper head, an iron shank and a wooden handle (Fig. 122).

High conductivity copper is almost exclusively used for the head, or bit, because of its high heat storage capacity and heat conductivity. Copper is also readily ' wetted ' with solder (tinned), and although the copper dissolves in molten solder this action is so slow that it does not hinder the free flow of the solder.

Size and shape. The size of the head is determined by the amount of heat which has to be stored in it and supplied to the job. It should always be sufficiently large to store enough heat to prevent the necessity for continual reheating. The tinned faces should be as large as possible to hold the maximum amount of solder without dripping.

The rate of heat delivery depends on the conductivity of the metals being soldered and the volume of metal within the heating range of the bit. The relative conductivity of a few metals, in descending order, is: silver, copper, aluminium, zinc, brass, tin, cast iron. Silver conducts heat away rapidly from the joint to the surrounding metal and therefore requires the supply of a great amount of heat to retain the soldering temperature at the joint. Cast iron, at the other end of the list, conducts the heat away slowly and little extra heat is required to make up any loss to the bulk of the metal.

Preheating parts to be soldered will reduce the heat conductivity loss. Heavy gauge metal parts are more difficult to solder because of the heat lost in bringing the mass of metal at the joint to the soldering temperature. Small parts should never be placed on metal which will take up a lot of heat, but on some insulating material such as wood. Wooden jaw pieces should be used when small parts are held in a vice.



FIGS. 122 and 123

The head shape for general soldering is square pointed. For repetition work, or awkwardly placed joints, other shapes are designed such as the hatchet head (Fig. 123).

The shank should be as short as possible to reduce fatigue, while being far enough away from the heat of the bit to prevent discomfort.

Preparing the bit. The bit will function properly only when it has been tinned. The faces are first cleaned with a rough file, then finished with a smooth file. Some sal ammoniac crystals (or a piece of fused zinc chloride) are placed on the hollow side of a brick. The bit is then heated until it will melt solder and rubbed on the sal ammoniac. This process is repeated, melting the solder, then rubbing on the sal ammoniac, until all the faces are tinned. The surplus solder is wiped off with a clean rag—never woollen as this will spoil the tinning. This procedure must be repeated when the faces become dirty in use.

Due to the working heat the copper bit rapidly oxidises and a scale is formed on it. The copper, as previously mentioned, dissolves in the molten solder and this causes pitting of the bit face. When this develops so that it interferes with the contact of the bit and work, the bit point should be filed to remove the pitting and then retinned.

Various metals have been tried to reduce this wear. Stainless steels are difficult to tin, readily lose the tinning and have too low heat conductivity. The non-corrosive copper-nickel alloys also have poor conductivity. It has been found however that nickel plating, spraying with aluminium or sherardising will improve the wearing quality of a copper bit.

The bit must not be overheated as the tin will be burned out of the solder.

Since the amount of heat conducted from the bit depends upon the contact area the bit face should be placed on the work and not just the point.

When vertical seams are to be soldered the bit is often tinned on one or two faces only. This prevents the solder running round the sides of the bit and dripping off the tip.

Continuously heated bit. A gas heated bit is heated by a jet which impinges on the back of the bit. Usually compressed air and high pressure gas are used and the bit is large enough to give a good heat storage capacity. Unfortunately these bits are often cumbersome because of the burner space and guard.

An electrically heated bit is heated by an element similar to an electric iron. It can be made quite small and is generally used on electrical and radio assembly work. For this work it is ideal, since the amount of solder required for each joint is small and the intermittant nature of the work allows time for the lost heat to be restored to the bit. A low voltage heater element is used to prevent any chance of electric shock. The electric bit is too slow for most long sheet metal joints.

Soldering by flame. When a soldered joint must be made with a minimum of heating of the article as a whole, a high temperature source of localised heat is used, that is, a bare flame. The essential requirements are that the time of application be very short and the flame be a small one. Considerable skill is required to direct a flame of exactly the right size where it is required, and it is essential that the workpieces be rigidly clamped or wired together. Solder can be applied from flux cored wire or as a paste of flux and solder.

Large flames or blow torches are used for filling in dents or weld marks on car bodies or other large parts.

Soldering Special Metals

Stainless and alloy steels. The common alloying elements in steels include chromium, nickel, manganese, molybdenum, tungsten and vanadium and these vary in amount up to 20 or 30 per cent.; one common stainless steel contains 18 per cent. chromium, 8 per cent. nickel. Today, ordinary mild steels are liable to contain small amounts of these alloying elements, introduced as scrap in making the steel, and may for this reason be difficult to solder.

The freshly exposed surface of certain alloy steels oxidises rapidly and the film of oxide prevents action between the steel and tin. To overcome this, the flux should contain hydrochloric acid in sufficient concentration to prevent this oxidisation of the surface. A suitable flux can be made by mixing equal quantities of commercial fused zinc chloride and hydrochloric acid. This flux is dangerous, giving off irritant fumes and spray which may cause corrosion. It is not suitable for production lines unless good ventilation is provided and special care is taken in washing the articles afterwards; the washing water should contain some soda and this should be followed by a plain water rinse.

High carbon steels are also difficult to tin. They are improved by prolonged pickling (15 minutes) in 20 per cent. nitric acid. Alternatively, the fused salt bath treatment as recommended for cast iron is effective.

Cast iron. The hard oxide skin on the casting must be removed by grinding, turning or shot blasting with small steel shot; sand blasting is much less effective and contaminates the surface with minute particles of sand which causes the tin coating to roll off. It is usually quite easy to tell by inspection the areas which are clear of the oxide skin.

After shot blasting, the parts are degreased in either trichlorethylene vapour or in boiling soda. Acid pickling is ineffective and leaves on the surface smears of graphite which resist tinning.

Immersion in a bath of molten flux salts at 270°C is effective provided the parts are held in the bath until they are heated sufficiently to melt solid flux on them. From the flux bath the articles are immersed in the solder pot which has a layer of molten flux on the surface of the solder. The flux recommended is:

Zinc chloride	75 per cent
Sodium chloride	15 per cent
Ammonium chloride	10 per cent

Soldering Copper Alloys

Brass and manganese bronze. Because we meet in everyday life so many brass parts which have been soldered we tend to think that the soldering of brass is a simple operation requiring little skill and technical knowledge. This is not so; the tin in the solder and the zinc in the brass do not readily alloy and this makes brass one of the less easy metals to solder. The higher copper content brasses, gilding metals, are more easily soldered than the yellow metals.

The brass may be cleaned by filing, scraping or rubbing with emery cloth or it may be pickled in cold hydrochloric acid (10 per cent.), or dipped in a hot bath of 2 per cent. nitric and 10 per cent. sulphuric acids. The part is now dried to prevent the flux flowing to where it is not wanted. The flux is: zinc chloride 9 parts, ammonium chloride 1 part, dissolved in 30 parts (by weight) of water. The solder generally used is 60/40. Antimony must not be present in the solder as it forms a brittle alloy of coarse crystals which weakens the joint considerably.

Bronze and Gunmetal can be readily soldered with a 60/40 solder; any reduction in the tin content makes soldering more difficult.

The whole process is similar to that described above.

Aluminium. Soldering this metal is difficult because there is no flux capable of removing efficiently at ordinary soldering temperatures the film of oxide, which forms again as fast as it is scraped away from the surface of the aluminium. This oxide film prevents the solder making contact with the bare aluminium and tinning it. The practical remedy for this difficulty is to arrange that the molten solder is already in position above the oxide film at the moment when it is scraped away, so that the tin in the solder can react before the air can do so. At the same time it is helpful to have present a cover of flux which will protect the solder itself from oxidising and also tend to lower the surface tension between the solder and aluminium and allow it to flow more freely.

In practice the aluminium surface is scraped bright and immediately coated with flux. Solder and a vigorous coal-gas or petrol blowtorch flame are applied together and as soon as the solder is molten the aluminium surface is scratched rapidly with a wire brush, hacksaw blade or file until the solder is seen to adhere where required. This operation can be carried out more easily if a jig is used to hold the work in the flame which is also held in the correct position. It is important to use a large enough flame so as to heat up the metal on either side of the joint and thus reduce the temperature loss due to the high heat conduction of aluminium. On the other hand, an intensely hot flame concentrated on the joint can give good results but, unless there is a substantial thickness of metal, there is a risk of burning through it; this risk can be reduced by backing the metal with a slab of steel while in the soldering jig.

The flux should exclude air; petroleum jelly is sometimes used, but it is rapidly burned away, and it is preferable to use a salt mixture which fuses easily and is capable of fluxing the solder. Examples are:

Potassium chloride	45 per cent
Sodium chloride	30 per cent
Lithium chloride	15 per cent
Potassium fluoride	7 per cent
Sodium pyrophosphate	3 per cent

or:

Barium chloride	40 per cent
Potassium chloride	30 per cent
Sodium chloride	20 per cent
Cryolite	10 per cent

These materials are mixed in finely powdered form and stored in sealed tins or jars and applied dry.

A suitable solder for aluminium contains 70 per cent. tin, 30 per cent. zinc; the melting temperature is about 320°C and it will flow readily if applied at 400°C or slightly less. The solder is sold in sticks a quarter of an inch in diameter, but smaller diameters, strip and powder forms are obtainable.

Soldered aluminium joints may corrode rapidly in damp conditions.

Fusable Metals as Solders

The combination of bismuth with tin and lead gives a range of alloys of remarkably low melting point and good soldering properties; they are called fusable metals. One such metal melts at 95°C and has the composition:

Bismuth	52 per cent
Lead	32 per cent
Tin	16 per cent

Metal to metal joints can be made with these metals at temperatures as low as 95°C. At these low temperatures there is no risk of the heat damaging foodstuffs, cloth or wood, nor will the temper of steel or bronze springs or sheet be affected.

Hot Tinning

Tin is a corrosion resistant, hygienic and readily solderable metal and hot tin coatings are applied for three main reasons.

(1) To provide hygienic protective coatings and give pleasing appearances to articles made from stronger and cheaper base metals.
(2) To facilitate the soldering of metals which are usually difficult.
(3) To assist the bonding of another metal to the base metal.

The usual thickness of coatings obtained by hot tinning is 0·0008 in. This imposes some limitations on the process since, for some purposes, thinner or thicker coatings may be required. These may be obtained by electrodeposition.

Hot tin coatings are usually produced by immersing the prepared work into a bath of molten tin, but they may also be formed by wiping, that is, applying the metal in molten stick or powder form to the prepared heated surface.

Bright lustrous surfaces are obtained direct from the tinpot and no polishing of the surface is required, although immersion in clean cold water is suggested when the metal cools to solidification.

All metals require cleaning and pickling before tinning is carried out and must then be smeared with a thin film of suitable flux. They should be slightly heated before dipping in the tinpot. Parts not requiring to be coated are covered with whitewash which acts as a protective coating. The tinning of mild steel has already been fully dealt with (p. 136). For other metals the following recommendations are made.

Copper. Pickling solution: dilute nitric acid, 15 per cent. in water.

Fluxes	Zinc chloride	24 lb
	Sodium chloride	6 lb
	Ammonium chloride	3 lb
	Hydrochloric acid	½ pint
	Water	10 galls.

In some cases palm oil or rosin act quite well on clean surfaces, but are not often used in dip tinning.

Dip tinning of copper. Only a small quantity of flux should cover the molten tin, which is maintained at a temperature of 250–260°C. The work should preferably be preheated to 200°C, at which temperature the flux covering should just begin to melt. The procedure for dipping is the same as for mild steel, the work being slowly immersed and rapidly withdrawn. Wiping is often carried out with pads of rag coated with tallow. The wiping is done while the coating is still fluid, to allow tin to redistribute itself over the surface and become as flat as possible.

Wipe tinning of copper. In wipe tinning the coating of tin is actually applied by a wiping process and this method is extensively used in the sheet metal industry for coating the insides of kettles, pans and boilers. The procedure used is also applicable to steel. The work is pickled, cleaned and placed over suitable heaters and a coating of flux is applied to surfaces to be treated. The tin is applied either by pouring the molten metal on to the job or in the form of a powder mixed with flux. Rubbing with a stick of tin is not suitable owing to the rapid formation of impurities on the surface. When the surface has been coated the surplus tin is wiped off and the work cooled. Any flux residues may be removed by rubbing with sawdust or wood meal.

The procedure adopted for copper work is also suitable for brass and bronze.

Cast iron is difficult to tin owing to the presence of carbon in the form of graphite on the surface of the metal. By scrupulous cleaning and pickling the metal may however be tinned using flux containing zinc chloride 9 oz., sodium chloride 2 oz. and water 2 pints. More than one coating of tin is required to give a bright clean surface.

Brazing

Brazing is carried out at a much higher temperature than that required for soft soldering. A borax type flux is used which fuses to allow brazing to take place between 750–900°C. There are a wide variety of alloys in use as brazing spelters, but in the main copper-zinc alloys are the most popular, and the following compositions can be fully recommended:

(1) Copper 59–61 per cent
(2) Copper 53–55 per cent } Remainder zinc.
(3) Copper 49–51 per cent

Impurities include tin, antimony, iron, arsenic, lead and bismuth. The total amount of impurities allowed in a brazing spelter is 0·75 per cent., and if this is exceeded the metal does not conform to standard specifications. Should a lower melting point be required, with increased fluidity, silver may be added in small quantities, and the metal is then known as silver solder.

Brazing spelter may be obtained in the form of wire or even in thin sheets which can be cut into strips of any desired width. It is also available in the form of small granules produced by passing the molten metal through an iron sieve and allowing the droplets so formed to fall into a water tank placed below the sieve and hence solidify.

Operation. The parts to be joined together are first cleaned with a file or scraper to remove all surface oxides, scale or grease. A mixture of borax and water paste is then painted on to the surfaces.

If the metal is clean and properly fluxed the molten spelter will follow the flux and penetrate right through the joint.

Heating of the joint area may be carried out by means of an oxy-acetylene blowpipe, a gas and air jet or a coke blast fire. If the latter is used the coke must be at a white heat before the job is placed on the fire, otherwise the metal will become contaminated with coke dust and will not allow the spelter to flow. As stated, the flux used for brazing is borax; this is sometimes mixed with other chemicals, but the only noticeable advantage of using a special flux is that the hard film of borate, which usually adheres to the surface of a brazed joint, does not form. This borate however does not affect the metal in any way, except that it is hard to remove by normal filing or scraping.

It is most important before proceeding to braze any article to ensure that the spelter has a lower melting point than the base metal, otherwise the work will be spoiled. Iron, steel, copper and brass are readily brazed and metals of a dissimilar nature can also be joined together, for example:

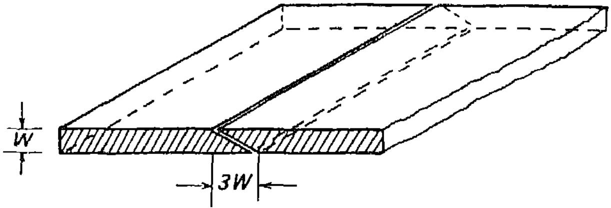
copper to brass
copper to steel
brass to steel
cast iron to mild steel.

Types of joints. Brazed joints made on sheet metal plates are most important in industry, for when correctly made it is possible to produce joints as strong or stronger than the solid plate. This is not possible in riveted work, where the strength varies from 55 per cent. in a single riveted joint to 80 per cent. for treble riveted joints.

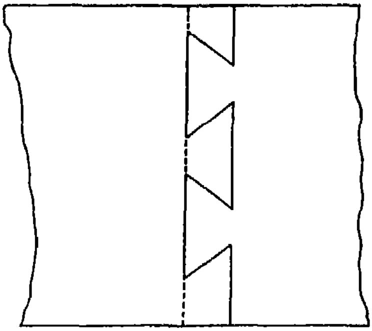
A *scarf* or wedge joint (Fig. 124(a)) is frequently used for low pressure steam pipes, water nozzles and also for making extensions to existing plates. Worked up bends for water columns are also jointed by these methods.

The edges of the plates to be joined are thinned down to form the scarf, as shown in Fig. 124(a). It has been proved by tests that the width of the overlap should be at least 3 times the plate thickness if strong joints are required. Brazing should be carried out with the plate horizontal, and heat should be supplied if possible from underneath the joint, thus allowing the molten metal to penetrate right through the joint.

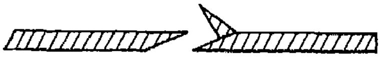
On heavier work, where strong unbreakable joints are required, a further extension to the scarf joint is made. This consists of thinning the edges of the metal as above, and then cutting the plate



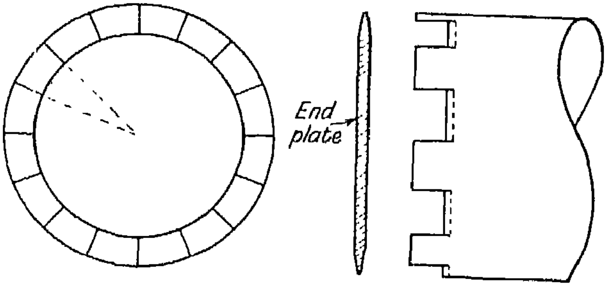
Scarf joint
(a)



Dovetailed joint



Section of plates before fitting
(b)



Dovetailed end for cylinder
(c)

FIG. 124

inwards in the form shown in Fig. 124(b). The name given to such a joint is a *dovetail.* One side only of the joint should be cut and under no circumstances is it permissible to remove any of the metal sheet. When this has been done the dovetails are bent alternately upwards and downwards, and the other uncut side of the joint is placed in position between them, each piece is cramped down to hold the joint together and then brazed. Dovetailed joints are used for fixing bottoms on pans and boilers; the method of fitting in the bottom plate is shown in Fig. 124(c).

This type of joint is usually only used for making joints in soft metals, such as copper and brass, and when completed the joint should be hammered on a solid bar and all surplus metal removed by filing or grinding.

TYPICAL EXAMINATION PAPER

SHEET METAL WORK

Written Paper

FIVE *questions only to be attempted.*

1. The base of a transformer is in the form of a rectangle 22 in. × 18 in. It terminates into a circle at the other end which is equal in cross-sectional area to the base. Calculate the diameter of the circular end. Ans. 22·46 in.
2. A cylindrical vessel is 9 in. diameter and 17½ in. high. Calculate its capacity in gallons. Take 1 gallon to equal 277·4 cu. in. Ans. 4·014 galls.
3. 200 pans of 1 ft 6 in. diameter were wired with No. 4 wire. Find the cost of wire used at the rate of 2*d.* per lb. Take 10 ft of No. 4 wire as weighing 1½ lb. Ans. £1 3*s.* 7*d.*
4. Which two metals are used when making soft solder? What percentage of each metal would you find in the following:
   (*a*) Plumbers' solder.
   (*b*) A solder used for dairy work?
   What would be the special properties of each type of solder?
5. Calculate the radius of the pattern for a right cone which is 21 in. diameter at the base and 25 in. vertical height. Ans. 27·1 in.
6. Define the difference between,
   (*a*) Physical and chemical changes,
   (*b*) Elements, mixtures and compounds.
7. A rectangular tank 6 ft × 4 ft × 4 ft deep is filled with oil. How many times will it fill a cylindrical container 10 in. diameter and 15 in. deep? Ans. 140 times.
8. 10 lengths of mild steel bar 3 in. wide by ½ in. thick and 12 ft long were bought for a job. Calculate the cost of this material at the rate of 18*s.* 8*d.* per cwt. Take 1 cu. in. of mild steel as weighing 0·28 lb. Ans. £5 0*s.* 9*d.*
9. A cylindrical vessel, without lid, is 3 ft diameter, 4 ft deep and made of 10 gauge mild steel. Assuming that there are no allowances for seams, calculate the weight of materials used in its construction. Take 10 gauge M/S as weighing 5 lb/sq. ft. Ans. 223·9 lb.

ANSWERS TO PROBLEMS

Exercise 1 (p. 5)

1. $\frac{39}{8}$; $\frac{38}{3}$; $\frac{59}{8}$; $\frac{53}{16}$; $\frac{299}{32}$; $\frac{129}{16}$.
2. $11\frac{1}{4}$; $2\frac{9}{10}$; $1\frac{5}{8}$; $1\frac{11}{16}$; $2\frac{17}{32}$; $1\frac{11}{64}$.
3. (*a*) $\frac{4}{16}$; $\frac{8}{16}$; $\frac{12}{16}$; $\frac{14}{16}$. (*b*) $\frac{2}{8}$; $\frac{4}{8}$; $\frac{6}{8}$. (*c*) $\frac{14}{64}$; $\frac{40}{64}$; $\frac{52}{64}$; $\frac{56}{64}$.
4. $\frac{1}{2}$; $\frac{3}{4}$; $\frac{1}{2}$; $\frac{3}{4}$; $\frac{7}{32}$; $\frac{5}{8}$.
5. $1\frac{3}{16}$; $\frac{19}{32}$; $\frac{29}{64}$; $\frac{7}{8}$; $1\frac{5}{16}$; $1\frac{15}{16}$.
6. $\frac{3}{8}$; $\frac{39}{64}$; $\frac{7}{16}$; $\frac{13}{16}$; $\frac{7}{16}$; $\frac{7}{8}$; $1\frac{1}{4}$; $5\frac{7}{8}$; $5\frac{5}{8}$; $5\frac{11}{16}$.
7. $\frac{9}{16}$; $\frac{3}{16}$; $1\frac{1}{32}$; $1\frac{9}{16}$; $\frac{3}{4}$; $\frac{15}{16}$; $2\frac{3}{16}$; $1\frac{27}{32}$; $4\frac{13}{16}$.
8. $4\frac{3}{16}$; $5\frac{1}{8}$; $4\frac{1}{16}$; $5\frac{3}{8}$; $9\frac{3}{16}$; $3\frac{15}{16}$.
9. $\frac{1}{4}$. **10.** $\frac{2}{3}$. **11.** $1\frac{7}{8}$ lb. **12.** $9\frac{3}{4}$ lb.
13. (*a*) $33\frac{1}{4}$ in. (*b*) 20 in. **14.** 3 in. **15.** $7\frac{3}{8}$ in.
16. $34\frac{5}{8}$ in. **17.** $9\frac{3}{8}$ in. **18.** $14\frac{3}{8}$ in. × $10\frac{1}{2}$ in. **19.** 5 in. × $3\frac{5}{16}$ in.
20. $19\frac{3}{4}$ hours. **21.** $2\frac{15}{16}$ in. **22.** 60 ft $5\frac{7}{8}$ in.
23. $20\frac{1}{8}$ in.

Exercise 2 (p. 10)

1. (*a*) $\frac{21}{128}$; (*b*) $13\frac{7}{9}$; (*c*) $25\frac{1}{3}$.
2. (*a*) $3\frac{23}{36}$. (*b*) $50\frac{61}{80}$. (*c*) $\frac{7}{64}$.
3. (*a*) $\frac{25}{32}$; (*b*) $12\frac{7}{84}$. (*c*) $9\frac{7}{30}$.
4. (*a*) $\frac{245}{284}$; (*b*) $12\frac{7}{8}$; (*c*) $9\frac{3}{16}$.
5. $3\frac{81}{112}$. **6.** $\frac{3}{4}$. **7.** 20. **8.** $\frac{3}{17}$. **9.** $\frac{60}{137}$.
10. $\frac{13}{180}$. **11.** 32 in. **12.** $\frac{9}{16}$ in. **13.** $28\frac{109}{112}$ in.
14. $5\frac{3}{4}$ in. **15.** £2 4*s.* $7\frac{1}{2}$*d.* **16.** men £7 1*s.* 5*d.* boy £2 7*s.* 2*d.*
17. 19 ft $5\frac{3}{4}$ in. **18.** $373\frac{3}{14}$ ft. **19.** copper $28\frac{63}{64}$ lb, zinc $13\frac{73}{80}$ lb, tin $3\frac{153}{320}$ lb.

Exercise 3 (p. 16)

1. (*a*) 38·9375; (*b*) 60·10783.
2. (*a*) 19; (*b*) 12; (*c*) 30,100.
3. (*a*) 0·002; (*b*) 1112; (*c*) 0·000008.
4. 0·078125, 11·28125, 7·5625, 0·734375.
5. 5·3125, 9·03125, 6·375, 3·142.
6. (*a*) 0·0012 in., 0·001044 in., 0·00189 in.
7. 46·08 lb. **8.** 7·65 lb. **9.** 5·484 lb.
10. (*a*) 4620 ft; (*b*) 3600.
11. (*a*) 2·58 in.; (*b*) 16; (*c*) 41·28 in.
12. 2251·4856 lb.

Exercise 4 (p. 19)

1. £4, £1 12*s.* 0*d.* **2.** 1·2 in.
3. 12·5 sq. in., 37·5 sq. in., 50 sq. in.
4. 5*s.* $7\frac{1}{2}$*d.* **5.** £20 14*s.* 7*d.* **6.** £2 5*s.* 0*d.*
7. £1 19*s.* 0*d.*, £2 12*s.* 0*d.*, £3 18*s.* 0*d.* **8.** 776 blanks.
9. Pressing; by £410 13*s.* 4*d.* **10.** £1 15*s.* 10*d.* **11.** Copper 61·6 lb., tin 7 lb., Zinc 1·4 lb. **12.** Copper 20 lb., Zinc 8 lb.

Exercise 5 (p. 22)

1. $8\frac{17}{40}$. **2.** 2*s.* 8*d.* **3.** 14·66. **4.** £13 4*s.* 0*d.*
5. 2836·512 lb. **6.** £26 9*s.* 9*d.* **7.** 27·89 per cent.
8. 0·8002 per cent. **9.** 171 gall. **10.** 83·28 per cent., 1 : 4·98.

Exercise 6 (p. 25)

1. (*a*) 625. (*b*) 6·25. (*c*) ·0625.
2. (*a*) $\frac{49}{64}$. (*b*) $6\frac{1}{8}$.
3. (*a*) 8. (*b*) 11. (*c*) 9.
4. (*a*) 46. (*b*) 213. (*c*) 5·3.
5. (*a*) $\frac{3}{8}$. (*b*) $\frac{3}{4}$. (*c*) $1\frac{1}{2}$.

Exercise 7 (p. 32)

1. (*a*) 1·8573, 0·8573, 4·8573. (*b*) 1, 2, 0.
(*c*) 0·8456, 4·3021, 2·9545. (*d*) 1·9035, 4·7166, 2·0026.
2. (*a*) 49·48, 4·948, 4948. (*b*) 1·007, 1000, 19·95.
(*c*) 405·4, 1·118, 3·057. (*d*) 670100, 97·5, 6·31.
3. (*a*) 7·446. (*b*) 3·468. (*c*) 28·02.
(*d*) 12·96. (*e*) 291700. (*f*) 664.
(*g*) 338. (*h*) 5249. (*i*) 776·9.
4. (*a*) 17·8. (*b*) 268·4. (*c*) 10·42.
(*d*) 42·63. (*e*) 8·312.
5. (*a*) 35·46. (*b*) 25·97. (*c*) 2·384.
(*d*) 2·144. (*e*) 16·42. (*f*) 1·26.
(*g*) 12. (*h*) 4·641. (*i*) 17·29.
6. (*a*) 1563. (*b*) 2·663. (*c*) 1·129.
(*d*) 1288. (*e*) 260·7. (*f*) 1953000.
7. (*a*) 2·6. (*b*) 179·5. (*c*) 1·972.
(*d*) 2·437. (*e*) 2·83. (*f*) 1·32.
8. (*a*) 4·767. (*b*) 1285. (*c*) 2·344.
(*d*) 9·27.
9. 5·23 ft. **10.** 3135 in.^{2} **11.** 47·7 cu. in.
12. 6·03. **13.** 638·5. **14.** 12·68.
15. 206. **16.** 1·297. **17.** 3310.
18. 26·33. **19.** 8·23. **20.** 34·36.

Exercise 8 (p. 38)

1. (i) $b = \frac{a-4}{h}$, $h = \frac{a-4}{b}$; (ii) $b = \frac{a}{h}$, $h = \frac{a}{b}$; (iii) $a = \frac{3V}{bc}$.

2. (i) $t = \frac{s}{2}$; (ii) $t = 2s$; (iii) $t = \frac{s}{v}$; (iv) $t = s - 2v$.

3. (i) $d = \frac{s+2}{a}$; (ii) $d = \frac{a+2}{4}$; (iii) $t = s - 3d$.

4. (i) $t = \frac{2s}{v}$; (ii) $t = \frac{dE}{l}$; (iii) $t = \frac{9h}{4p}$; (iv) $t = \frac{5bR}{6a}$

5. $t = \sqrt{\frac{2s}{g}}$. **6.** $r = \sqrt{\frac{V}{\pi h}}$. **7.** $d = \sqrt{\frac{TD}{pl}}$. **8.** $V = \sqrt{PR}$

9. (i) $L = \frac{P}{ft}$; (ii) $t = \frac{P}{Lf}$; (iii) $f = \frac{P}{Lt}$, 30·7 in.

10. (i) $V = \frac{428t^2}{P}$; (ii) $t = \sqrt{\frac{PV}{428}}$, 0·1 in.

11. 0·043 in. **12.** 3·45 in.

Exercise 9 (p. 51)

1. 6·58 sq. ft. **2.** (*a*) 708·9 lb.; (*b*) 21·23 per cent.
3. 27·1 per cent. **4.** 1·833 per cent. **5.** 116·96 lb.
6. 6·068 in. **7.** 23·86 in. radius. **8.** $R = 7{\cdot}15$ ft.; $r = 3{\cdot}9$ ft.
9. 27·2 lb; 42·4 in. **10.** 50·65 ft.

Exercise 10 (p. 60)

1. 9 ft 8½ in. **2.** 2·98 ft. sides. **3.** 15·08 revs.
4. (*a*) 3·385 lb.; (*b*) 60 8 per cent. **5.** 4·733 in.
6. R = 14·71 in.; θ = 110°; Circumference of base = 28·26 in.
7. 4·898 in. diam. **8.** 19·536 in. long, 9·768 in. wide.
9. 7·77 in. **10.** (*a*) 15·03 in.; 10·23 in.; 13·54 in.; (*b*) 60·12 in.; 32·07 in.; 42·5 in.

Exercise 11 (p. 70)

1. 755. **2.** (*a*) 4 ft 0·15 in.; (*b*) 59·37 sq. ft.
3. (*a*) 2·412 lb.; (*b*) 5·58 per cent; **4.** (*a*) 14 in.; (*b*) 175·8 lb.
5. 3313 cu. in.; 2152 cu. in. **6.** 278·7 miles.
7. 176·7 yds. **8.** 190·8 cu. in.; 5·152 lb.
9. 7·64 in. **10.** 117·4 lb.

Exercise 12(a) (p. 78)

1. 82·94 cu. in.; 83·45 sq. in.
2. 2·12 gallons; 15·47 lb.
3. 13·32 galls.; 133·2 lb.
4. 360·7 lb. **5.** 60 times. **6.** 3·582 in. **7.** 387·1 cu. in.
8. 0·02 in. **9.** 33·09 galls. **10.** 14·91 cu. ft.

Miscellaneous Exercises (p. 94)

1. 2·06 ft dia. **2.** 408 sq. in. **3.** 4771 gall. **4.** 3·56 in.
5. 943·65 sq. ft. **6.** 5·33 in. dia. **7.** 302 cu. in., 188·5 sq. in.
8. 125 days. **9.** 25 per cent. **10.** 46½ per cent. **11.** 107·29 sq. in.
12. 4·319 in. **13.** 15·17 in., 41·16 lb. **14.** 13·86 in. dia.
15. 9·9 in. dia., 8·774 in. length of side. **16.** 27·71 lb, 8·802 gall.
17. 224 cu. ft. **18.** 35·1 sq. ft. **19.** 26·055 in. by 8·685 in., 13·54 in. dia, 10·36 in. dia.
20. 4·9 in. **21.** (*a*) 4·45 cu. ft.; (*b*) 27·81 gall. **22.** (*a*) 16·5 cu. ft.; (*b*) 103·125 gall.
23. 117·45 cu. in. **24.** 230·69 cu. in.

Exercise 13 (p. 105)

1. (i) 0·8863; (ii) 3·0473; (iii) 0·2357.
2. (i) 0·4171; (ii) 0·6839; (iii) 0·9716.
3. (i) 0·4452; (ii) 0·9753; (iii) 0·2829.
4. (i) 65° 42′; (ii) 48° 14′; (iii) 69° 31′.
5. (i) 6° 19′; (ii) 64° 47′; (iii) 17° 34′.
6. (i) 48° 32′; (ii) 78° 8′; (iii) 15° 20′.
7. (i) 10 in.; (ii) 10 in. (iii) 70° 55′.
8. (i) 1·589; (ii) 6·472 in.; (iii) 34° 51′.
9. 1·732 in. **10.** 10·98 in., 9 in. **11.** 104·9 in. **12.** 88·59 lb.
13. 60·57 in.; 15·19 in. **14.** 23·64 lb. **15.** (*a*) 15 in.; (*b*) 11·18 in.
16. 97° 10′; 22 in.

Exercise 14 (p. 113)

1. 3 ton/in^2. **2.** 4·32 ton. **3.** 181 lb/in^2.
4. 13·75 ton. **5.** 0·5x lb. **6.** 122·5 lb.
7. 78·75 lb. **8.** V.R. = 62·9:1; Effort = 10·5 lb.
9. V.R. = 32:1; M.A. = $20\frac{5}{6}$; Efficiency = 65·1 %
10. 770 in.; $6\frac{1}{7}$ lb.

TABLES

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

NATURAL SINES

°	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	14
5	·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	11	14
10	·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	11	14
11	·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	·2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	3	6	9	11	14
13	·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	13
23	·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	13
24	·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	10	12
36	·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

NATURAL SINES

°	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
45	·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	·7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	4	4
73	·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	1
85	·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	0	0
88	·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	·9998	9999	9999	9999	9999	1·000	1·000	1·000	1·000	1·000	0	0	0	0	0

NATURAL COSINES

SUBTRACT

°	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
45	·7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10
46	·6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11
47	·6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
48	·6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
49	·6561	6547	6534	6521	6508	6494	6481	6468	6455	6441	2	4	7	9	11
50	·6428	6414	6401	6388	6374	6361	6347	6334	6320	6307	2	4	7	9	11
51	·6293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
52	·6157	6143	6129	6115	6101	6088	6074	6060	6046	6032	2	5	7	9	12
53	·6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	2	5	7	9	12
54	·5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
55	·5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
56	·5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
57	·5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
58	·5299	5284	5270	5255	5240	5225	5210	5195	5180	5165	2	5	7	10	12
59	·5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	13
60	·5000	4985	4970	4955	4939	4924	4909	4894	4879	4863	3	5	8	10	13
61	·4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
62	·4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
63	·4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
64	·4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
65	·4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
66	·4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	13
67	·3907	3891	3875	3859	3843	3827	3811	3795	3778	3762	3	5	8	11	13
68	·3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
69	·3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
70	·3420	3404	3387	3371	3355	3338	3322	3305	3289	3272	3	5	8	11	14
71	·3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
72	·3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
73	·2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
74	·2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
75	·2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
76	·2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
77	·2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
78	·2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
79	·1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	11	14
80	·1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	11	14
81	·1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3	6	9	12	14
82	·1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3	6	9	12	14
83	·1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
84	·1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
85	·0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	14
86	·0698	0680	0663	0645	0628	0610	0593	0576	0558	0541	3	6	9	12	15
87	·0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
88	·0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
89	·0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9	12	15

SUBTRACT

NATURAL COSINES

SUBTRACT

°	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
0	1·000	1·000	1·000	1·000	1 000	1 000	**·9999**	**9999**	**9999**	**9998**	0	0	0	0	0
1	·9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
2	·9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	0	0
3	·9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0	0	1	1	1
4	·9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0	0	1	1	1
5	·9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	1
6	·9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
7	·9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
8	·9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
9	·9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
10	·9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
11	·9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1	1	2	2	3
12	·9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
13	·9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
14	·9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
15	·9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
16	·9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
17	·9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	4	4
18	·9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1	2	3	4	5
19	·9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
20	·9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
21	·9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
22	·9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
23	·9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1	2	3	5	6
24	·9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	4	5	6
25	·9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
26	·8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
27	·8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	7
28	·8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
29	·8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
30	·8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1	3	4	6	7
31	·8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
32	·8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
33	·8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2	3	5	6	8
34	·8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
35	·8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
36	·8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2	3	5	7	9
37	·7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
38	·7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
39	·7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
40	·7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
41	·7547	7536	7524	7513	7501	7490	7478	7466	7455	7443	2	4	6	8	10
42	·7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
43	·7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
44	·7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10

Figures in bold type show change of integer

SUBTRACT

NATURAL TANGENTS

°	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

NATURAL TANGENTS

°	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′
45	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	26	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1·9626	9711	9797	9883	9970	**0057**	**0145**	**0233**	**0323**	**0413**	15	29	44	58	73
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	118
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	130
71	2·9042	9208	9375	9544	9714	9887	**0061**	**0237**	**0415**	**0595**	29	58	87	116	145
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	203
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	Use interpolation				
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972					
77	4·3315	3662	4015	4373	4737	5107	5483	5864	6252	6646					
78	4·7046	7453	7867	8288	8716	9152	9594	**0045**	**0504**	**0970**					
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80	5·6713	7297	7894	8502	9124	9758	**0405**	**1066**	**1742**	**2432**					
81	6·3138	3859	4596	5350	6122	6912	7720	8548	9395	**0204**					
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	**0285**					
83	8·1443	2636	3863	5126	6427	7769	9152	**0579**	**2052**	**3572**					
84	9·5144	6768	8448	10·02	10·20	10·39	10·58	10·78	10·99	11·20					
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95					
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46					
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27					
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08					
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0					

Figures in bold type show changes of integer